Financial Econometrics[*]

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This Revision: October 1, 2006

Abstract

This is an introduction to a five-volume collection of papers on financial econometrics to be published by Edward Elgar Publishers in 2007. Financial econometrics is one of the fastest growing branches of economics today, both in academia and in industry. The increasing sophistication of financial models requires equally sophisticated methods for their empirical implementation, and in recent years financial econometricians have stepped up to the challenge. The toolkit of financial econometrics has grown in size and depth, including techniques such as nonparametric estimation, functional central limit theory, nonlinear time-series models, artificial neural networks, and Markov Chain Monte Carlo methods. In these five volumes, the most influential papers of financial econometrics have been collected, spanning four decades and five distinct subfields: statistical models of asset returns (Volume I), static asset-pricing models (Volume II), dynamic asset-pricing models (Volume III), continuous-time methods and market microstructure (Volume IV), and statistical methods and non-standard finance (Volume V). Within each volume, different strands of the literature are weaved together to form a rich and coherent historical perspective on empirical and methodological breakthroughs in financial markets, while covering the major themes of financial econometrics.

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[*] I thank John Cox and John Heaton for helpful discussions.

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General Introduction

As a discipline, financial econometrics is still in its infancy, and from some economists' perspective, not a separate discipline at all. However, this is changing rapidly, as the publication of these volumes illustrates. The growing sophistication of financial models requires equally sophisticated methods for their empirical implementation, within academia and in industry, and in recent years financial econometricians have stepped up to the challenge. Indeed, the demand for financial econometricians by investment banks and other financial institutions—not to mention economics departments, business schools, and financial engineering programs throughout the world—has never been greater. Moreover, the toolkit of financial econometrics has also grown in size and sophistication, including techniques such as nonparametric estimation, functional central limit theory, nonlinear time-series models, artificial neural networks, and Markov Chain Monte Carlo methods.

What can explain the remarkable growth and activity of this seemingly small subset of econometrics, which is itself a rather esoteric subset of economics?

The answer lies in the confluence of three parallel developments in the last half century. The first is the fact that the financial system has become more complex over time, not less. This is an obvious consequence of general economic growth and development in which the number of market participants, the variety of financial transactions, and the sums involved have also grown. As the financial system becomes more complex, the benefits of more highly developed financial technology become greater and greater and, ultimately, indispensable.

The second factor is, of course, the set of breakthroughs in the quantitative modeling of financial markets, e.g., financial technology. Pioneered over the past three decades by the giants of financial economics—Fischer Black, John Cox, Eugene Fama, John Lintner, Harry Markowitz, Robert Merton, Franco Modigliani, Merton Miller, Stephen Ross, Paul Samuelson, Myron Scholes, William Sharpe, and others—their contributions laid the remarkably durable foundations on which all of modern quantitative financial analysis is built. Financial econometrics is only one of several intellectual progeny that they have sired.

The third factor is a contemporaneous set of breakthroughs in computer technology, including hardware, software, and data collection and organization. Without these computational innovations, much of the financial technology developed over the past fifty years would be irrelevant academic musings, condemned to the dusty oblivion of unread finance journals in university library basements. The advent of inexpensive and powerful desktop microcomputers and machine-readable real-time and historical data breathed life into financial econometrics, irrevocably changing the way finance is practiced and taught. Concepts like alpha, beta, R^2, correlations, and cumulative average residuals have become concrete objects to be estimated and actively used in making financial decisions. The outcome was nothing short of a new industrial revolution in which the "old-boys network" was replaced by the computer network, where what mattered more was *what* you knew, not *who* you knew, and where graduates of Harvard and Yale suddenly found themselves at a disadvantage to graduates of MIT and Caltech. It was, in short, the revenge of the nerds!

But there is an even deeper reason for the intellectual cornucopia that has characterized financial econometrics in recent years—it is the fact that randomness is central to both finance and econometrics. Unlike other fields of economics, finance is intellectually vapid in the absence of uncertainty; the net-present value rule and interest-rate compounding formulas are the only major ideas of non-stochastic finance. It is only when return is accompanied by risk that financial analysis becomes interesting, and the same can be said for econometrics. In contrast to many econometric applications where a particular theory is empirically tested by linearizing one of its key equations and then slapping on an error term as an afterthought, the sources and nature of uncertainty are at the core of every financial application. In fact, the error term in financial econometrics is the main attraction, not merely a disturbance to be minimized or averaged away. This approach creates a rich tapestry of models and methods that have genuine practical value because the randomness assumed is more closely related to the randomness observed than in other econometric applications. Indeed, the econometric consequences of uncertainty in financial models usually follow directly from the economics, and are not merely incidental to the empirical analysis.

The papers collected in Volume I, which consists of the most influential statistical models of financial-asset returns, is the starting point for this intimate connection between finance and econometrics. Even without the economic infrastructure of preferences, supply and demand, and general equilibrium, the contributions in Volume I shed considerable light on the basic properties of asset returns such as return predictability, fat tails, serial correlation, and time-varying volatilities.

The papers in Volume II are able to extract additional information from asset returns and volume by imposing additional structure, e.g., specific investor preferences, parametric probability distributions for underlying sources of uncertainty, and general equilibrium. Using a two-period or static framework—the simplest possible context in which price uncertainty exists—the papers of Volume II yield remarkably simple yet far-reaching implications for the relation between risk and expected return, the proper economic definition of risk, and methods for evaluating the performance of portfolio managers.

Of course, the static two-period framework is only an approximation to the more complex multi-period case, which is the focus of the articles in Volume III. By modeling the intertemporal consumption/savings and investment decisions of investors, a wealth of additional testable implications can be derived for the time-series and cross-sectional properties of asset returns and volume.

The dynamics of prices and quantities lead naturally to questions about the fine structure of financial transactions and markets, as well as the notion of continuous-time trading, both of which are examined by the articles of Volume IV. The econometrics of continuous-time stochastic processes is essential for applications of derivatives pricing models to data, and the practical relevance of continuous-time approximations is dictated by the particular market microstructure of the derivative's underlying asset.

Volume V is the final volume of the series and contains methodological papers, as well as contributions to finance that are not yet part of the mainstream, but which address important issues nonetheless. In particular, this volume includes papers on quantifying selection and data-snooping biases in tests of financial asset-pricing models, Bayesian methods, event-study analysis, Generalized Method of Moments estimation, technical analysis, neural networks, and some examples from the emerging field of econophysics.

The sheer breadth of topics across the five volumes should give readers a sense of the impact and intellectual vitality of financial econometrics today. Moreover, the articles in these volumes also span a period of four decades—ranging from classic tests of the Random Walk Hypothesis in the 1960's to the application of random matrix theory to portfolio optimization in 2002—illustrating the remarkable progress that the field has achieved over time. Along with the many innovations produced by global financial markets will be a never-ending supply of wonderful challenges and conundrums for financial econometrics, guaranteeing its importance to economists and investors alike.

I Statistical Models of Asset Returns

Ever since the publication in 1900 of Louis Bachelier's thesis in which he modelled stock prices on the Paris Bourse as Brownian motion, finance and statistics have become inextricably linked. In Part I of Volume I, we begin with four articles that provide some much-needed philosophical background for the role of statistical inference in financial modeling. While statistics now enjoys an independent existence, replete with general and specialized journals, conferences, and professional societies, the financial econometrician has a somewhat different perspective. The uniqueness of financial econometrics lies in the wonderful interplay between financial models and statistical inference, where neither one dominates the other. In particular, **Cox (1990)** underscores the importance of models that guide the course of our statistical investigations, but **Leamer (1983)**, **McCloskey and Ziliak (1996)**, and **Roll (1988)** provide some counterweight to the economist's natural tendency to depend more on models than on facts.[1]

The intimate relationship between financial theory and statistical properties is illustrated perfectly by the Random Walk Hypothesis, which is the subject of the articles in Part II. Unlike the motivation for Brownian motion in physics and biology—the absence of information—the economic justification for randomness in financial asset prices is active information-gathering on the part of all market participants. It is only through the concerted efforts of many investors attempting to forecast asset returns that asset returns become unforecastable. This leads to several testable implications, and much of the early literature in financial econometrics consisted of formal statistical tests of the Random Walk Hypothesis and corresponding empirical results (**Working, 1960**; **Mandelbrot, 1963**; **Fama, 1965**; **Lo and MacKinlay, 1988**; **Poterba and Summers, 1988**; **Richardson and Stock, 1990**).

However, randomness is not the only interesting characteristic of financial asset returns. Many authors have documented a host of empirical properties unique to financial time series including time-varying moments (**Engle, 1982**; **Bollerslev, 1986**; **Nelson, 1991**), fat tails

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[1] References in boldface are included in the *Financial Econometrics* volumes.

and long-range dependence (**Mandelbrot and Van Ness, 1968**; **Greene and Fielitz, 1977**; **Granger and Joyeux, 1980**; **Geweke and Porter-Hudak, 1983**; **Lo, 1991**; **Baillie,1996**), regime shifts (**Hamilton, 1989**), and in some cases, co-integrated price processes (**Engle and Granger, 1987**; **Phillips, 1987**). Each of these issues is addressed in Parts III–V of Volume I through a series of specific stochastic processes designed to capture these properties, along with empirical evidence that either supports or rejects these models for financial data.

Collectively, the papers in Volume I should provide readers with a comprehensive arsenal of statistical descriptions of financial time series, all motivated by particular empirical observations.

II Static Asset-Pricing Models

The focus of the previous volume was the statistical properties of financial asset-returns, without reference to any specific economic model of investors or financial interactions. In Volume II, we shift our attention to the relative magnitudes of asset returns over a given time period. On average, is the return to one stock or portfolio higher than the return to another stock or portfolio, and if so, to what can we attribute the difference?

These questions are central to financial economics since they bear directly on potential trade-offs between risk and expected return, one of the most basic principles of modern financial theory. This theory suggests that lower-risk investments such as bonds or utility stocks will yield lower returns on average than riskier investments such airline or technology stocks, which accords well with common business sense: investors require a greater incentive to bear more risk, and this incentive manifests itself in higher expected returns. The issue, then, is whether the profits of successful investment strategies can be attributed to the presence of higher risks—if so, then the profits are compensation for risk-bearing capacity and nothing unusual; if not, then further investigation is warranted. In short, we need a risk/reward benchmark to tell us how much risk is required for a given level of expected return. The first, and perhaps most celebrated financial model that provides an explicit risk/reward trade-off for financial asset returns is the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). In the CAPM framework, an asset's "beta" is the relevant measure of risk—stocks with higher betas should earn higher returns on average. And in many of the recent anomaly studies, the authors argue forcefully that differences in beta cannot fully explain the magnitudes of return differences, hence the term "anomaly".

The articles in Part I of this volume provide a comprehensive analysis of the CAPM, and chronicles a fascinating intellectual journey that begins with simple but elegant tests of the CAPM that find support for the theory (**Fama and MacBeth, 1973**), leading to more sophisticated statistical tests of the CAPM (**Gibbons, 1982**; **Jobson and Korkie, 1982**; **MacKinlay, 1987**; **Gibbons, Ross, and Shanken, 1989**), and ends with serious questions about the explanatory power of the CAPM versus to other multi-factor models (**Fama and French, 1992**; **Black, 1993**; **MacKinlay, 1995**; **Lo and Wang, 2000**). But the historical significance of this literature goes well beyond the CAPM—this line of inquiry was the first to employ rigorous statistical inference, ushering empirical finance into the modern age of financial econometrics.

Part II contains a parallel stream of the Arbitrage Pricing Theory (APT) literature in financial econometrics. Despite the fact that the APT might seem like a close cousin of the CAPM—both are, after all, linear factor models of asset returns—the empirical APT literature was, for a time, stuck in a theoretical quagmire in which the falsifiability of the APT was questioned (**Shanken, 1982, 1985**; **Dybvig and Ross, 1985**). While a cynic might argue that the best theory is one that can never be disproved, respectable scientific mores suggest otherwise, and the sometimes-bitter debate surrounding this issue yielded many nuggets of theoretical (**Chamberlain and Rothschild, 1983**), econometric (**Connor and Korajczyk, 1993**), and empirical (**Dhrymes, Friend, and Gultekin, 1984**; **Roll and Ross, 1984**; **Chen, Roll, and Ross, 1986**; **Lehmann and Modest, 1988**) wisdom for the profession.

An important outgrowth of the many econometric innovations surrounding the empirical analysis of the CAPM and APT is the performance attribution literature, the focus of the articles in Part III. It is a truism that one cannot manage what one cannot measure, hence it should come as no surprise that the proper measurement of performance has become an essential part of investment management. In particular, measures of security-selection ability (**Treynor and Black, 1973**) and market-timing ability (**Merton, 1981**; **Henriksson and Merton, 1981**), and statistical inference for risk/reward measures such as the Sharpe ratio (**Lo, 2002**; **Getmansky, Lo, and Makarov, 2004**) have now become part of the practitioner's lexicon in discussing investment performance. This is another example of how academic research in financial econometrics has made an indelible impact on financial practice.

III Dynamic Asset-Pricing Models

The static asset-pricing models of Volume II are clearly meant to be approximations to a more complex reality in which investors and financial markets interact through time. The challenges of dynamic asset-pricing models are considerable, since they involve many more degrees of freedom for market participants and security prices. It is far easier to model the conditional distribution of tomorrow's stock price than the joint distribution of daily prices over the next five years. However, by imposing sufficient structure on investor preferences and security-price dynamics, it is possible to develop a rich yet testable theory of asset prices over time.

The articles in Part I of Volume III illustrate this possibility through the variance-bounds test of market rationality. By assuming that a security's market price is equal to the capitalized value of all future payouts, and by assuming that payouts follow a stationary stochastic process, it is possible to derive an upper bound for the variance of that security's price based on the subsequent stream of payouts. The empirical fact that this variance bound is apparently violated by aggregate historical U.S. equity prices has been interpreted as a violation of market rationality (**LeRoy and Porter, 1981**; **Shiller, 1981**), a conclusion with far-reaching implications for all kinds of financial decisions if it were true. This observation added fuel to the already-smoldering debate between proponents of market rationality and its critics, yielding enormously valuable insights into the econometrics of equilibrium asset prices. For example, by replacing the assumption of stationarity for prices and payouts with the Random Walk Hypothesis—which is arguably closer to empirical reality and theoretical consistency (recall Bachelier's model of stock prices on the Paris Bourse)—the upper bound becomes a lower bound, i.e., the inequality is reversed (**Marsh and Merton, 1986**). Also, because of estimation error, the empirical violation of the variance bound may be attributed to sampling fluctuation (**Flavin, 1983**; **Kleidon, 1986**; **West, 1988**).

But one of the most interesting outcomes of the variance bounds literature is its implications for the sociology of scientific inquiry in economics and finance. Like a magnet dropped into a dish of iron filings, the variance bounds debate polarized the academic community almost immediately, with members of economics departments lining up behind the irrationalists, and members of finance departments in business schools taking the side of market rationality. The debate should have been settled by the weight of econometric analysis and empirical fact, but remarkably, with each new publication that peeled back another layer of this wonderfully controversial challenge, the convictions of the disciples in both camps only grew stronger. To this day, there is no consensus; the response to the title of **Shiller's (1981)** paper "Do stock prices move too much to be justified by subsequent dividends?", is "yes" if you teach in an economics department and "no" if you teach in a business school.

The variance bounds controversy had another salutary effect on the financial econometrics literature: its focus on aggregate measures sparked additional interest in asset-pricing models based on aggregate measures of consumption. This, in turn, led to a number of significant breakthroughs in asset-pricing theory and econometrics, including the equity premium puzzle (**Mehra and Prescott, 1985**), consumption-based asset-pricing models (**Hansen and Singleton, 1983**; **Breeden, Gibbons, and Litzenberger, 1989**), stochastic discount factor models (**Hansen and Jagannathan, 1992**), and asset-pricing models with incomplete markets (**Heaton and Lucas, 1996**) and state-dependent preferences (**Campbell and Cochrane, 1999**). Although these models have met with limited empirical success, they have generated an enormous literature at the intersection of macroeconomics and finance, enriching our understanding of both in the process.

In Part III, we turn our attention from stock markets to bond markets. Bonds, particularly default-free government bonds, are inherently simpler financial instruments because unlike the dividend streams paid by equity securities, the nominal cashflows of bonds are prespecified and nonstochastic. There are only three major sources of uncertainty affecting bond prices: interest rates or discount rates over various horizons, realized and expected inflation, and the probability of default. Addressing the first source of uncertainty is the motivation for models of the term structure of interest rates, and one of the earliest models employed curve-fitting techniques to the data (**McCulloch, 1971**; **Vasicek and Fong, 1982**). But the most influential term structure model is the celebrated Cox, Ingersoll, and Ross (1985) model, a dynamic general equilibrium model that incorporates investor preferences among other aspects of the macroeconomy. Although empirical implementations of this model has yielded mixed results (**Brown and Dybvig, 1986**; **Gibbons and Ramaswamy, 1993**), it has served as the durable foundation of an extensive literature of more econometrically oriented models of the term structure (**Duffie and Singleton, 1993, 1997**).

The two remaining sources of uncertainty for bond prices—inflation and default—have rich literatures of their own, much of which is beyond the scope of this series but which has been summarized in other series. Two examples of that literature have been included in Part III for completeness (**Fama and Bliss, 1987**; **Lo 1986**).

IV Continuous-Time Methods and Market Microstructure

One of the great ironies of modern economics is the fact that most of its theories assume that individuals take prices as given, yet the primary objective is usually to explain how prices are determined. In an economy where everyone takes prices as given, how do prices change? The nineteenth-century mathematical economist Léon Walras hypothesized the existence of an auctioneer who calls out a price, observes the excess demand or supply generated by that price, and then adjusts the price up or down so as to reduce the excess demand or supply. Although a figment of the economist's imagination, this process of "tatonnement" was perhaps the first systematic attempt to model the price-discovery process. A more careful examination of how prices are set—from one transaction to the next—has yielded a number of important insights into the fine structure of economic interactions, and this is the purview of the market microstructure literature.

Although market interactions have been the subject of virtually all economic analysis since the publication of Adam Smith's (1776) *An Inquiry into the Nature and Causes of the Wealth of Nations*, market microstructure phenomena are distinct. For example, the impact of price discreteness (**Ball, 1988**; **Hausman, Lo, and MacKinlay, 1990**), irregular trading intervals (**Scholes and Williams, 1977**; **Dimson, 1979**; **Cohen et al. 1983**; **Lo and MacKinlay, 1990**), and the bid/offer spread (**Roll, 1984**; **Glosten and Harris, 1988**) have only recently been studied thanks to the growing interest in the microstructure of financial markets. Much of this literature owes its genesis to the availability of machine-readable transactions-level data, pioneered by Robert A. Wood and first analyzed in **Wood, McInish, and Ord (1985)**. Since then, other transactions datasets have become available through organized exchanges such as the New York Stock Exchange (the TAQ and TORQ datasets) or through brokerage firms such as Investment Technology Group (limit-order data, analyzed in **Lo, MacKinlay, and Zhang, 2002**).

The irregular timing of trades at the transaction level highlight the fact that standard discrete-time models cannot fully accommodate the richness of financial markets. This provides part of the motivation for articles in Part II and the set of continuous-time models in the finance literature, in which prices are assumed to evolve continuously through time, typically with sample paths that are everywhere continuous. However, continuous sample paths imply that price movements are smooth, which, in turn, implies that over infinitesimally short time intervals, price changes are completely forecastable. Such an implication is clearly at odds with both reality and basic finance theory—perfectly forecastable prices would mean either unlimited profit opportunities (buy low, sell high), or nonstochastic prices which brings us back to the trivial case of financial markets with no uncertainty.

This conundrum was first recognized and addressed by the French mathematician Louis Bachelier in his 1900 doctoral thesis on warrant pricing (see Volume I), who developed the basic notions of Brownian motion, a continuous-time version of the Random Walk Hypothesis. This mathematical object is strange indeed—it is a continuous-time sequence of random variables, where every sample path is continuous, but because even infinitesimal increments are unforecastable, the sample paths are nowhere differentiable. In other words, sample paths are continuous but so jagged that it is impossible to compute its rate of change even over the smallest time intervals. Without the pioneering insights of Bachelier, Albert Einstein (who developed a similar model in 1905 while studying the photoelectric effect), and Nobert Wiener (who was the first to develop a rigorous mathematical formulation of Brownian motion), it is not obvious that a continuous-time random walk should exist.

But it does, and Brownian motion has become the workhorse of modern financial economics thanks to a related mathematical breakthrough achieved by the mathematician Kiyosi Itô (1951): the stochastic calculus and a corresponding theory of stochastic differential equations. The importance of this innovation to modern finance is elegantly described in **Merton**

(1975). Briefly, the assumption of continuous-time trading, coupled with the ability to derive the exact laws of motion for nonlinear functions of Brownian motion via stochastic calculus, implies that it is possible to replicate the payoffs of complex financial instruments such as options and other derivative securities by dynamically adjusting portfolios of simpler securities such as stocks and bonds. This insight, first hypothesized by Arrow (1964), was given substance in Merton's (1973) option-pricing model which, along with Black and Scholes (1973), revolutionized financial theory and practice by providing not only exact pricing models for derivative securities, but also explicit methods for hedging and synthetically manufacturing such securities through dynamic trading strategies.

These ideas, for which Merton and Scholes shared the 1997 Nobel Prize, have led to numerous breakthroughs in finance theory, but remarkably, they are also partly responsible for the birth of at least three distinct multi-trillion-dollar businesses in the finance industry: organized options exchanges (e.g., the Chicago Board Options Exchange, the International Securities Exchange, the Boston Options Exchange), the over-the-counter derivatives business (e.g., caps, floors, collars, swaptions, etc.), and today's burgeoning credit-derivatives business (e.g., credit default swaps, CDS swaptions, credit-linked notes).

A pre-requisite to any application of continuous-time models in financial markets is to estimate the parameters of the stochastic differential equations that describes the prices of the underlying securities. This creates another challenge for financial econometrics: the econometrics of continuous-time stochastic processes. The fact that time is hypothesized to be continuous, yet in practice we observe data only at discrete, and sometimes irregular, time intervals, causes a number of difficulties that are not present in discrete-time models. These issues are addressed in **Clark (1973)**, **Garman and Klass (1980)**, **Parkinson (1980)**, **Shiller and Perron (1985)**, and **Lo (1986, 1988)**. The root of many of these issues is the subtle relationship between discrete and continuous time—both are, after all, approximations to reality.

The nature of these approximations is derived explicitly by **Bertsimas, Kogan, and Lo (2000)**, where they reverse the standard chain of logic by using option-pricing models to define the notion of "temporal granularity" and to measure the discrepancies between discrete and continuous time. A number of other econometric applications have arisen from the derivatives pricing literature, including implied binomial trees (**Rubinstein, 1994**), nonparametric estimation of state-price densities and risk aversion (**Aït-Sahalia and Lo, 1998, 2000**), and semi-parametric bounds for option prices (**Lo, 1987**).

As useful as Brownian motion is, it cannot capture all aspects of the data, two of which are particularly relevant for financial data: price jumps, and serial correlation. Fortunately, a number of alternatives exist for modeling both in continuous time, including **Ball and Torous (1985)** and **Lo and Wang (1995)**.

Finally, the use of continuous-time stochastic processes in modeling financial markets has led, directly and indirectly, to a number of statistical applications in which functional central limit theory and the notion of *weak convergence* (see, for example, Billingsley, 1968) are used to deduce the asymptotic properties of various estimators, of which **Richardson and Stock (1990)** in Volume I is an excellent example.

V Statistical Methods and Non-Standard Finance

As discussed in the General Introduction, one of the most attractive characteristics of finance from the econometrician's perspective is the central role that uncertainty plays. Therefore, it should come as no surprise that statistical inference and financial models are intimately connected. In Volume V, we have collected a host of important contributions to financial econometrics that are primarily methodological in focus, though each article is motivated by a particular financial challenge.

Perhaps the most basic challenge to empirical work in finance is the wealth of data available to financial econometrics, and the many false positives that can result from repeated analysis of such data. It is no exaggeration that if one tortures a dataset long enough, it will confess to anything! In Part I, the magnitude of this phenomenon—also known as selection bias, data-snooping bias, and backtest bias—is investigated both analytically and numerically. **Brown et al. (1992)** conclude that survivorship bias—the bias induced by including only the surviving corporations or mutual funds in an empirical study—can be quite substantial. **Lo and MacKinlay (1990)**, and **Foster, Smith, and Whaley (1997)** come to similar conclusions for other forms of selection biases, and propose new statistical methods to adjust for such biases.

One of the most significant sources of bias is the pre-conceived notions that empirical researchers cling to as they formulate their experimental designs. Perhaps the only systematic approach to taking these pre-conceived notions or "priors" into account is to use a Bayesian statistical framework for conducting inferences, as in the articles of Part II. Bayesian methods are widely used in the statistics literature, hence they can be applied to almost any context in which statistical inference is called for. In the financial context, Bayesian methods have been applied to portfolio optimization (**Klein and Bawa, 1977**; **Shanken, 1987**; **Kandel, McCulloch, and Stambaugh, 1995**) and tests of the Arbitrage Pricing Theory (**McCulloch and Rossi, 1991**) and other asset-pricing models (**Harvey and Zhou, 1990**). Although historically quite cumbersome and, as a result, not particularly appealing to the mainstream financial econometrics literature, recent advances in computationally intensive methods for conducting Bayesian inferences, e.g., the Gibbs sampler and Markov Chain Monte Carlo methods, has revolutionized the field.

The articles in Part III contain other important techniques that any practicing financial econometrician should be familiar with, including event studies (**Fama et al. 1969**; **Brown and Warner, 1985)**; **Ball and Torous, 1988**), Generalized Method of Moments (**Hansen, 1982**; **Richardson and Smith, 1991**), and robust methods for estimating expected returns and covariance matrices (**Merton, 1980**; **Newey and West, 1987**).

The last part of Volume V, and the final set of articles for the *Financial Econometrics* series, is a collection drawn from the underbelly of mainstream finance. Part IV includes articles on nonlinear dynamical systems (**Hsieh, 1991**), neural networks (**Hutchinson, Lo, and Poggio, 1994**), technical analysis (**Brock, Lakonishok, and LeBaron, 1992**; **Lo, Mamaysky, and Wang, 2000**), and random matrix theory (**Pafka and Kondor, 2002**; **Plerou et al. 1999**). Although they are not yet part of the mainstream, they have generated sufficient interest in either academia or industry to deserve inclusion in these volumes. For example, while technical analysis—the practice of forecasting future price movements based on geometric patterns in historical price plots—is generally dismissed by the academic finance community, it is the arguably the most widely used set of techniques among commodities and foreign exchange traders. Similarly, random matrix theory is a set of mathematical results originally developed by nuclear physicists for modeling statistical fluctuations of particle interactions, but has recently been applied with some degree of success to estimating covariance matrices for portfolio optimization problems.

These are only a very small and idiosyncratic sample of the tremendously rich and varied literatures that are loosely connected in one way or another to financial econometrics. Although they are at the outskirts of the finance literature today, they may well join the mainstream in the near future once they prove their practical worth.

Concluding Thoughts

Two decades ago, the term "financial econometrics" did not exist. It is a remarkable testament to the practical value of financial econometrics that we have been able to fill five volumes with pathbreaking articles in this nascent discipline. In contrast to other branches of economics, e.g., industrial organization, labor economics, and macroeconomics, the application of econometric analysis to financial markets has given birth a new and cohesive field of study. Yet the list of unanswered research questions is still much longer than the list of achievements that financial econometrics has produced so far. For example:

- How do we conduct proper statistical inference for financial time series, which are usually non-stationary, non-Gaussian, skewed, leptokurtic, and neither independently nor identically distributed?
- How do we decide which portfolio managers have skill when the standard errors of the usual performance statistics are so large that over 500 years of monthly returns are required to yield any kind of statistical significance?
- Is there a way to adjust simulated portfolio returns to account for backtest bias?
- What is the best way to measure the likelihood of rare events and manage such risks if, by definition, there are so few events in the historical record?
- How should we construct optimal portfolios of securities if estimated means and covariance matrices are subject to so much estimation error?
- How can we estimate the risk preferences of an individual or institutional investor, and are these preferences stable over time and individuals?

- Is the extraordinary investment performance of certain portfolio managers due to their extraordinary risk exposures, or does genuine alpha exist in the investment management business?

These questions are surprisingly simple to state, yet so far no consensus has been reached as to how to answer any of them. They are just a few of the wonderful challenges that lie in store for future generations of financial econometricians.

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