A brief introduction to

Chebfun

By Nick Hale

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Philosophy:

Numerical computing with functions.

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“Computing with symbolic feel and numerical speed”

Symbolic Computing (Maple, Mathematica, etc.)

Manipulate formulas exactly.
When you want numbers, evaluate the formulas.

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$$
\begin{aligned}
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(SymPy fails...?)

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Numerical Computing (MATLAB, C, Fortran, etc.)

Work with numerical approximations instead of exact expressions.
Perform each operation to relative accuracy of about 10^{-16}.
This kills the combinatorial explosion.

PROBLEM: What if we want not just numbers, but functions like f(x)?

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Plan:

Overload standard MATLAB vector routines
with continuous (1D) analogues.

Implementation:

Machine precision interpolation with
Chebshev polynomials.

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!= Hybrid symbolic/numeric computing.

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The software:

Version 4.1 release 01/08/2011.
BSD(new) license.
Written in MATLAB.
Nightly builds (www.chebfun.org/nightly/)

Feel free to download and follow along!

This talk:

A brief introduction via some demos.
Description of one or two core routines.
Examples of some more advanced features.

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The Project

+ Started in 2006, open-source in 2011 with v4.0
+ ~2500 downloads (+MWFE) since v4.0 release in March '11
+ ~15 contributors? (Still mostly in Oxford...)
+ SVN for version control and Trac for bug reports/wiki
+ ~1000 M-files & ~60,000 lines of code
+ ~20-100 citations? (It's hard to count!)
+ ~100 online Examples (I'll show you some later!)

The People

Matlab Demo

How it works

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Function evaluations of f at Chebyshev nodes

↓ FFT

$$f(x) \approx \sum c_k T_k(x)$$

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Function evaluations of f at Chebyshev nodes

$$\downarrow \text{FFT}$$

$$f(x) \approx \sum c_k T_k(x)$$

+ $T_k(x) = \cos(k \,\mathrm{acos}(x)) \Rightarrow |T_k(x)| \leqslant 1$
+ $f \in C^d[-1,1] \Rightarrow c_k = \mathcal{O}(k^d)$, $f \in H[-1,1] \Rightarrow c_k = \mathcal{O}(e^{-Ck})$

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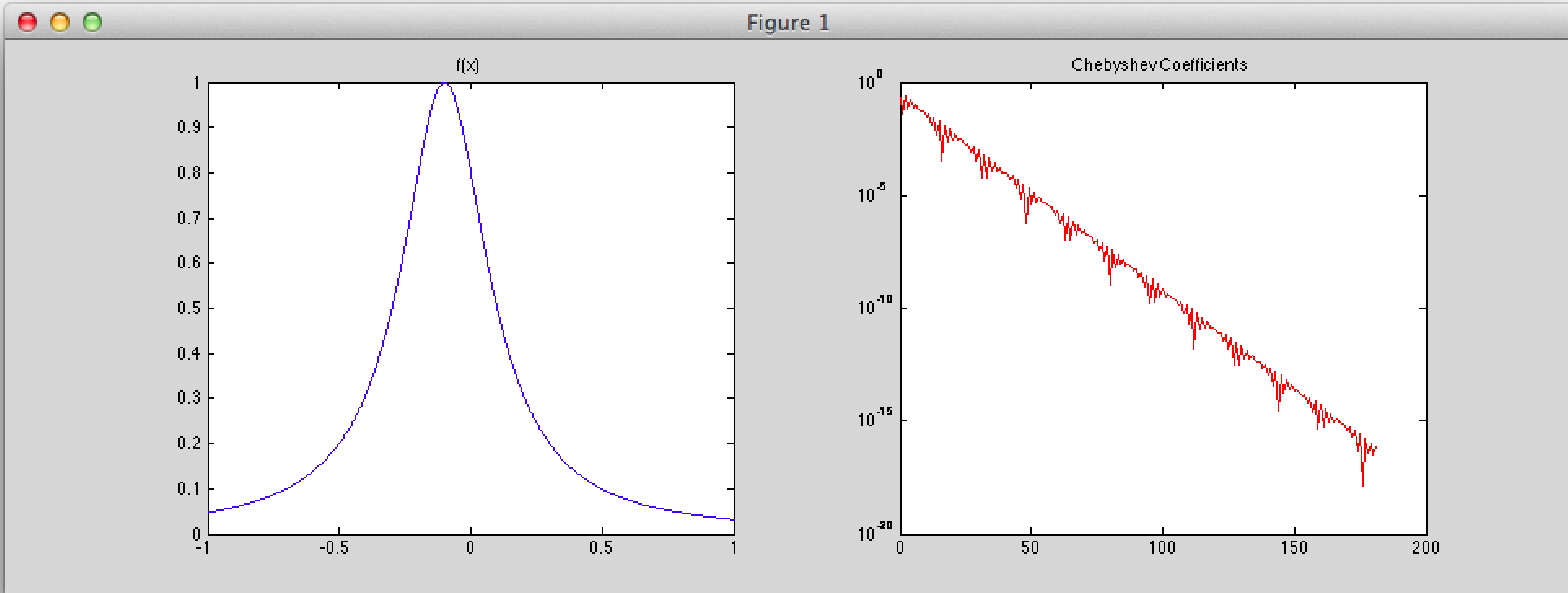
Algorithm:

1. Interpolate at n+1 Chebyshev points.
2. Convert function values to coefficients.
3. Converged? No $\rightarrow$ increase n & repeat,
   Yes $\rightarrow$ done.

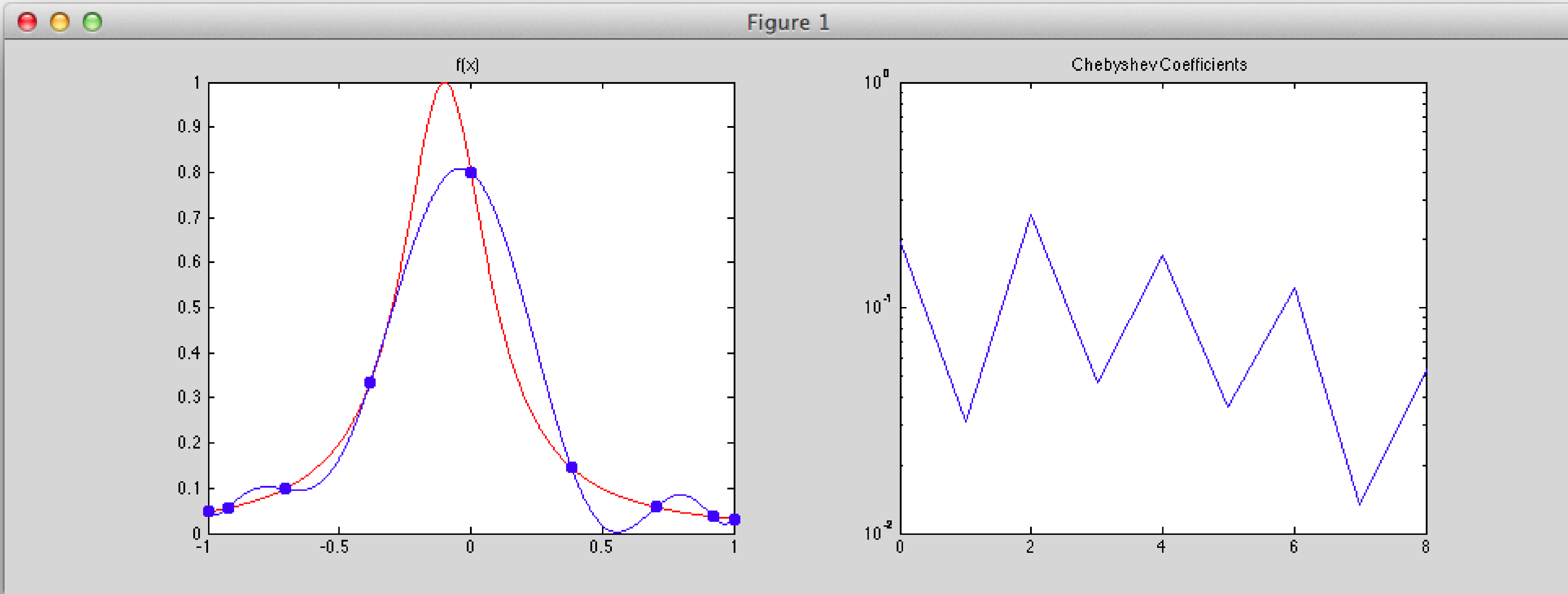
How it works

```
>> f = chebfun(@(x) 1./(1+25*(x+.1).^2));
>> chebpolyplot(f);
```



How it works

```
>> f = chebfun(@(x) 1./(1+25*(x+.1).^2), 9);
>> chebpolyplot(f);
```



How it works

```
>> f = chebfun(@(x) 1./(1+25*(x+.1).^2), 17);
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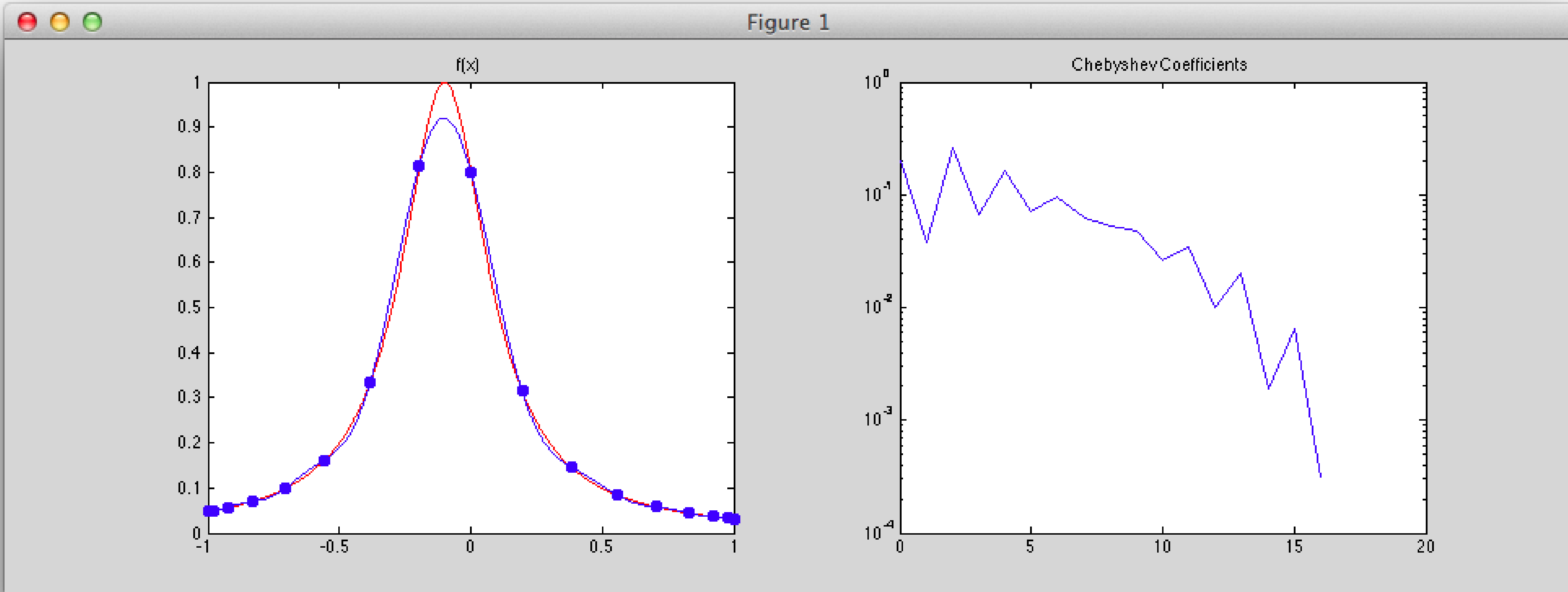

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>> f = chebfun(@(x) 1./(1+25*(x+.1).^2), 33);
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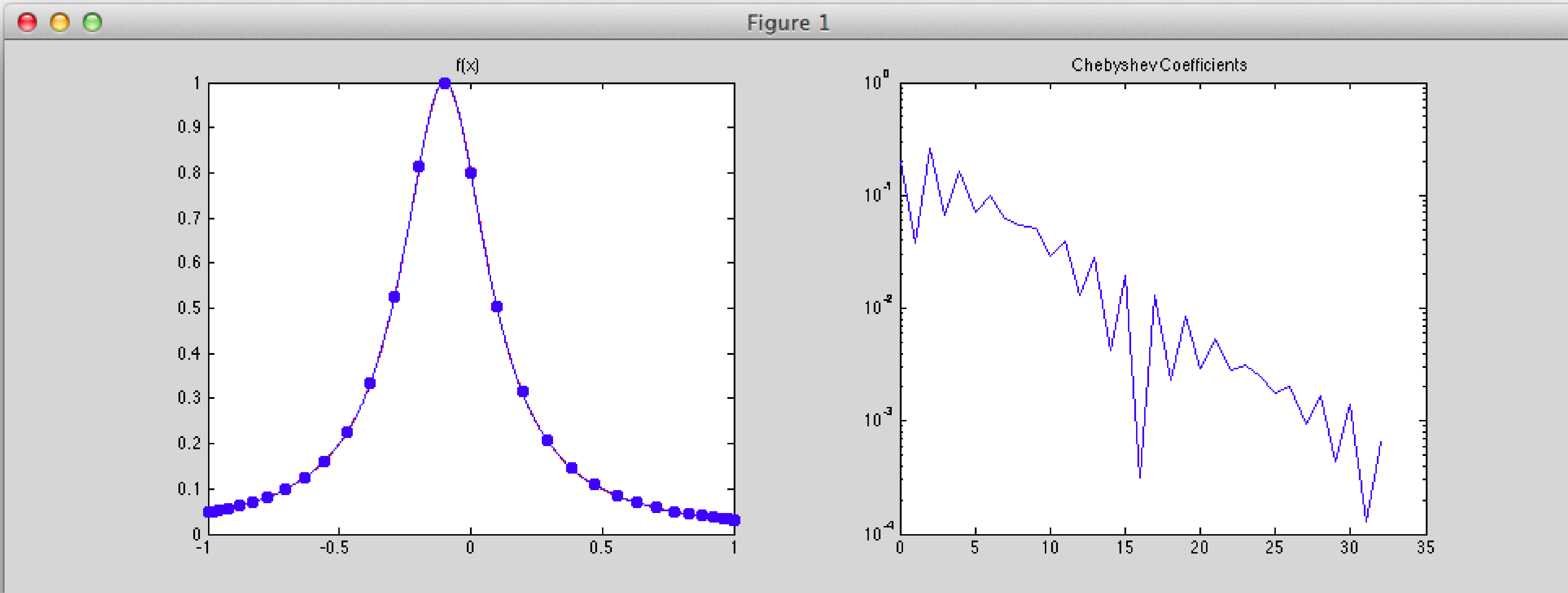
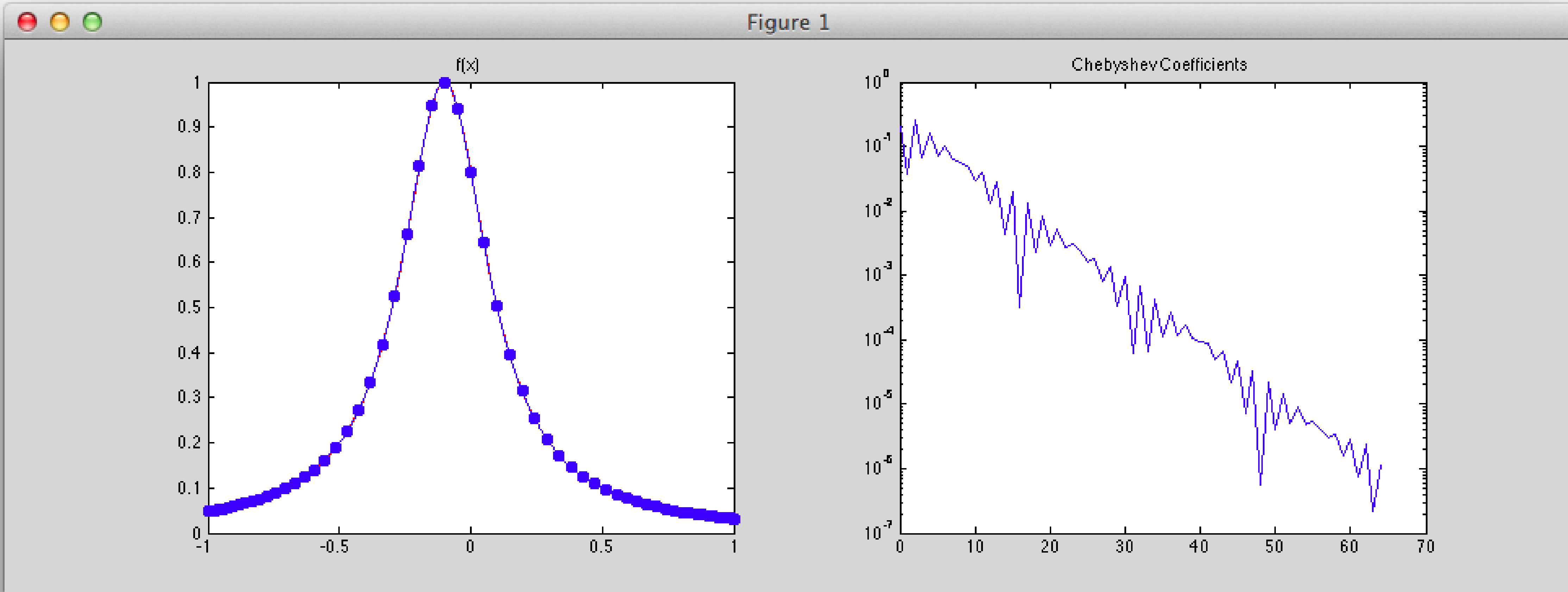


How it works

```
>> f = chebfun(@(x) 1./(1+25*(x+.1).^2), 65);
>> chebpolyplot(f);
```



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>> f = chebfun(@(x) 1./(1+25*(x+.1).^2), 129);
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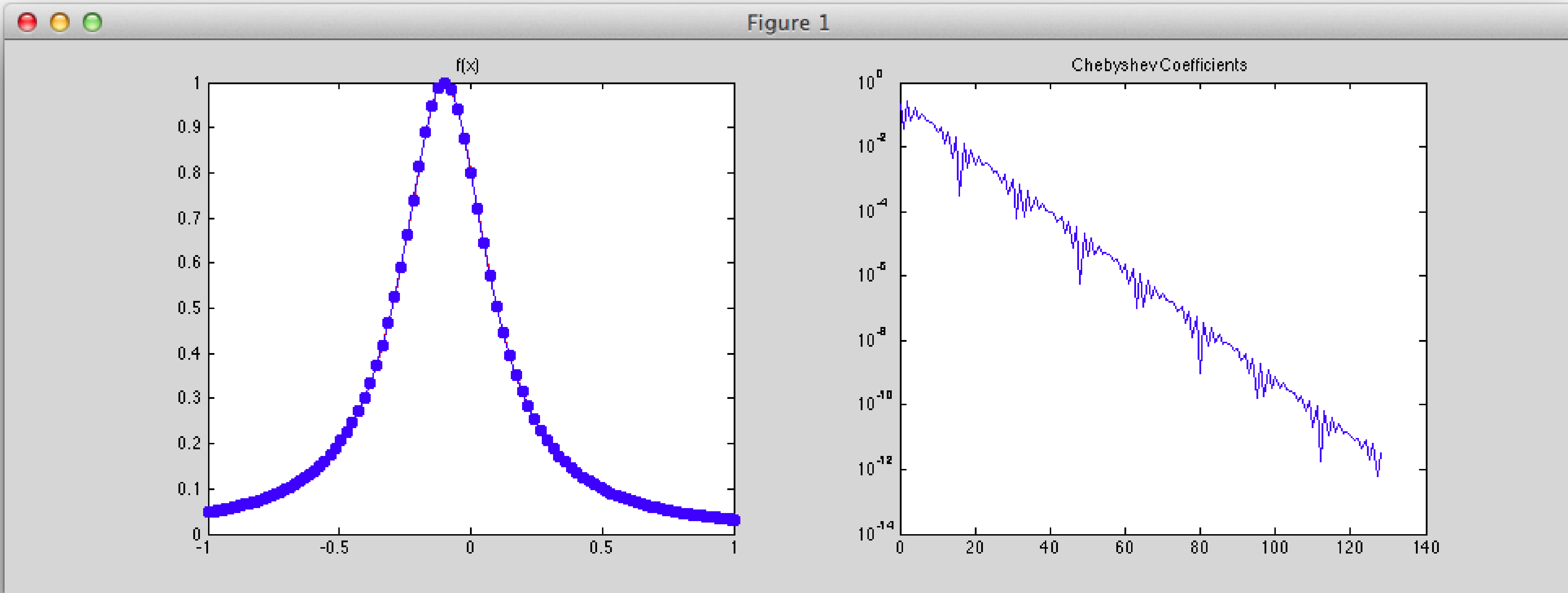

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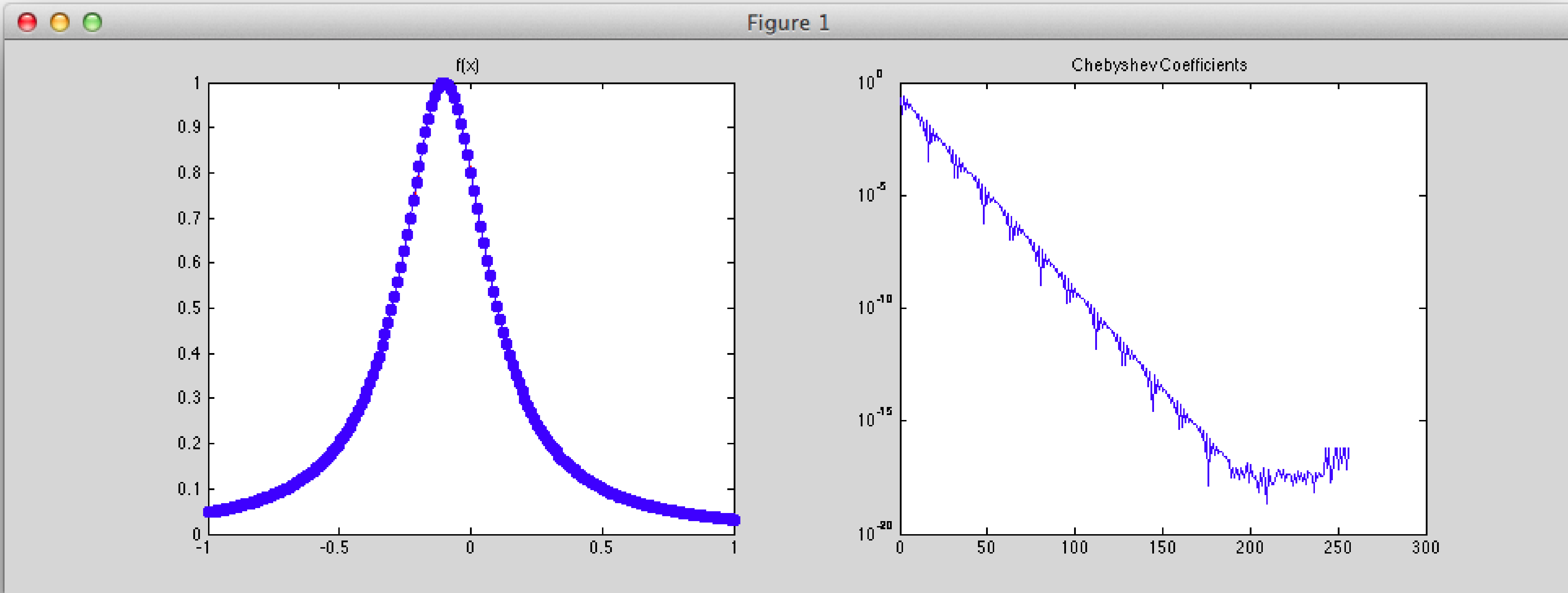

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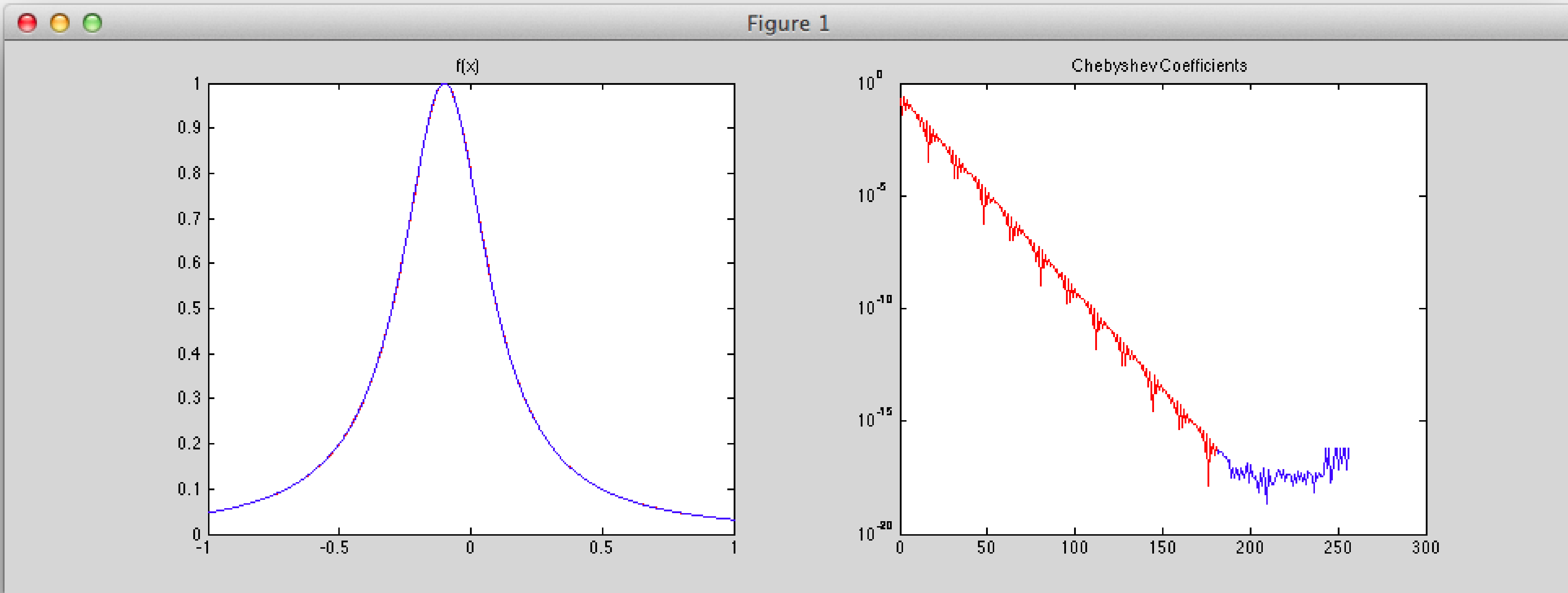

How it works (cont.)

+ Evaluation → Barycentric formula
+ Integration → Clenshaw-Curtis quadrature
+ Differentiation → Recurrence on coefficients
+ Rootfinding → Colleague matrix of coefficients

Differential Eqns

Chebyshev Spectral Methods
(One slide introduction)

$$f(x) = p_n(x) \Rightarrow f'(x) \approx p_n'(x)$$

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$$0.1u'' + u' + xu = 1$$

$$(0.1D_n^2 + D_n + \mathrm{diag}(\underline{x}))p_n(\underline{x}) = L_n p_n(\underline{x}) = \underline{1}$$

$$u(\underline{x}) \approx p_n(\underline{x}) = L_n \backslash \underline{1}$$

(Plus some boundary conditions...)

Chebyshev Spectral Methods
(One slide introduction)

$$L_n \, u_n = f_n$$

+ Check for happiness in u
+ If not happy, increase n
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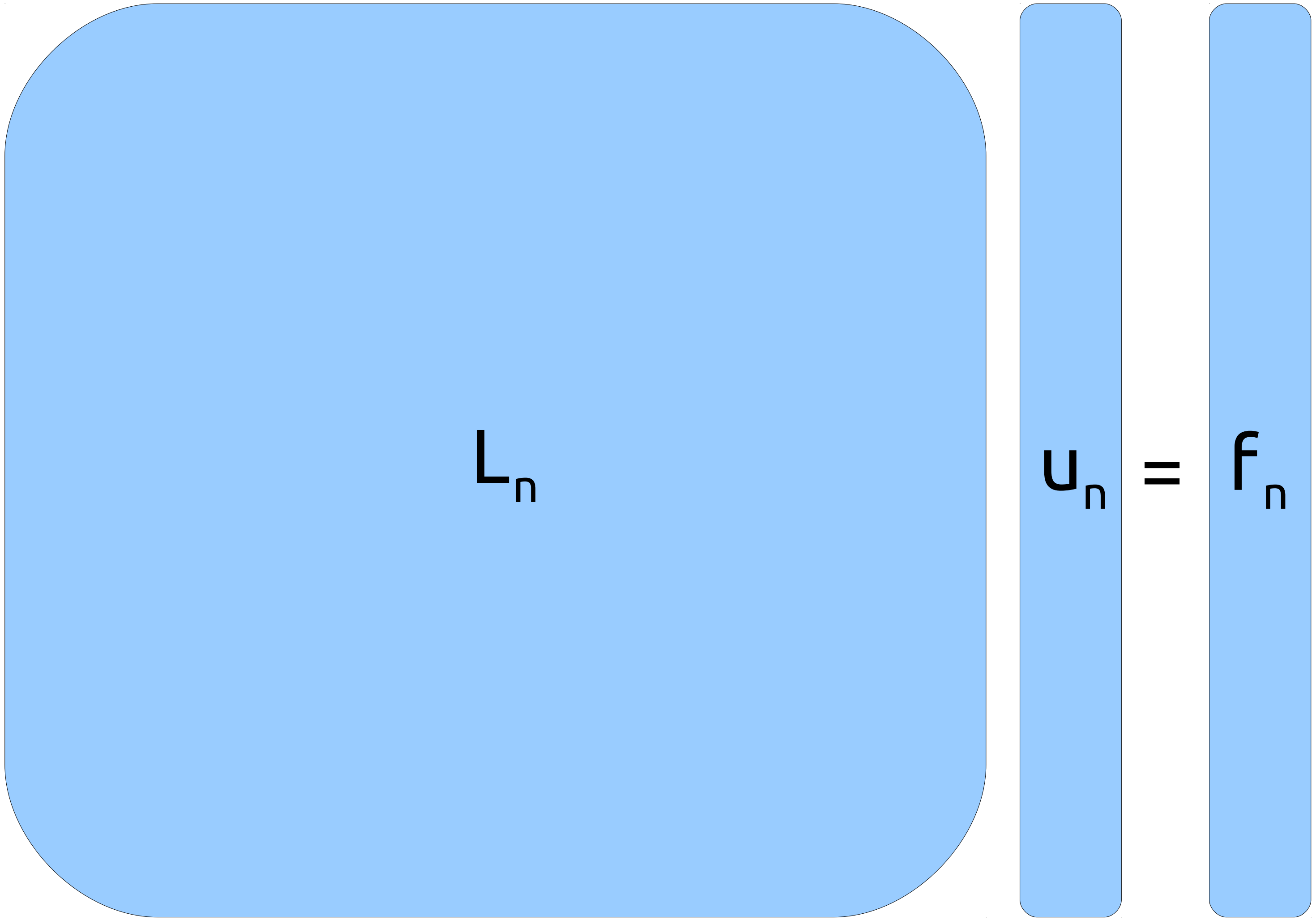

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Nonlinear ODEs, N(u,x) = 0.
(Newton iteration)

+ Extend idea of rootfinding via Newton method to continuous framework & solve linear subproblems.

$$u \longleftarrow u - \text{diff}(N(u,x),u)\backslash N(u,x)$$

+ Requires (Fréchet) derivatives of the operators involved, which are obtained by Automatic Differentiation (AD).

MATLAB Demo

Examples

Eigenvalue Repulsion

If you morph one NxN matrix A into the another B by the formula

$$C(t) = (1-t)A + tB,$$

then as $t : 0 \rightarrow 1$, the eigenvalues change continuously from those of A to those of B .

The phenomenon of “level repulsion”, or “eigenvalue avoided crossings”, goes back to von Neumann and Wigner, and states that with probability 1 there is no t for which C has a multiple eigenvalue.

We can verify this in Chebfun!

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```
n = 10;
A = randn(n); A = A+A';
B = randn(n); B = B+B';
ek = @(e,k) e(k);              % returns kth element of the vector e
eigA = @(A) sort(eig(A));      % returns sorted eigenvalues of the matrix A
eigk = @(A,k) ek(eigA(A),k); % returns kth eigenvalue of the matrix A
for k = 1:n
   E(:,k) = chebfun(@(t) eigk((1-t)*A+t*B,k),[0 1]);
end
plot(E)
```

Eigenvalue Repulsion

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then as t : 0 →

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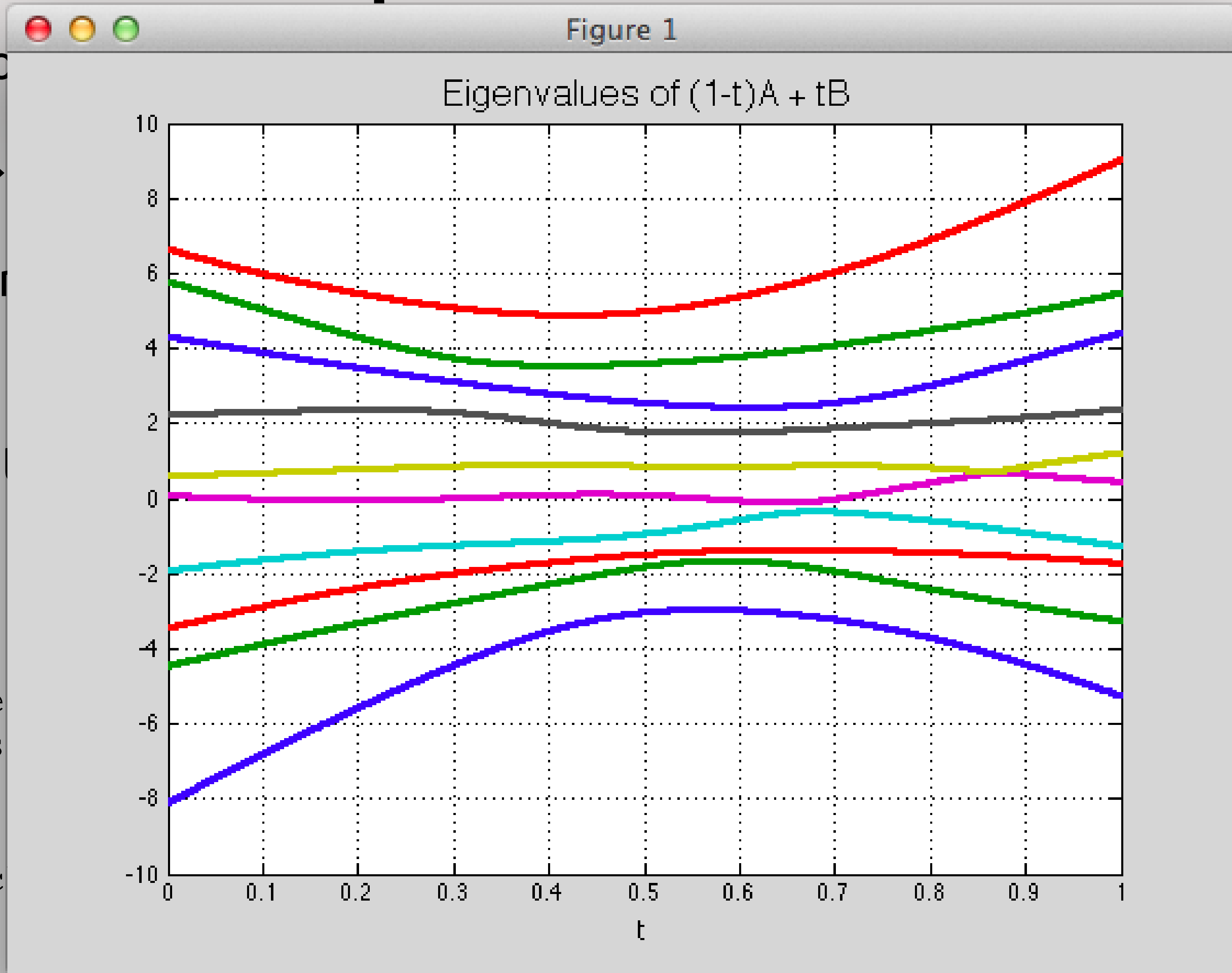


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ek = @(e,k) e                                       e
eigA = @(A) s                                       matrix A
eigk = @(A,k)                                       ix A
for k = 1:n
   E(:,k) = c
end
plot(E)
```

Optics: Eigenvalues of Fox-Li

In the field of optics, integral operators arise that have a complex symmetric (but non-Hermitian) oscillatory kernel. An example is the following linear Fredholm operator L, associated with the names of Fox and Li:

$$Lu(x) = v(x) = \sqrt{iF/\pi} \int_{-1}^{1} K(x,s) u(s) ds$$

L maps a function u defined on [-1,1] to another function v = Lu defined on [-1,1]. The number F is a positive real parameter, the Fresnel number, and the kernel function K(x,s) is

$$K(x,s) = \exp(-iF(x-s)^2)$$

Compute the 80 largest eigenvalues of L.

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```
F = 64*pi;                                 % Fresnel number
K = @(x,s) exp(-1i*F*(x-s).^2 );           % Kernel
L = sqrt(1i*F/pi)*fred(K,domain(-1,1));    % Fredholm integral operator
lam = eigs(L,80,'lm');                     % Compute eigenvalues
plot(lam);                                 % Plot
```

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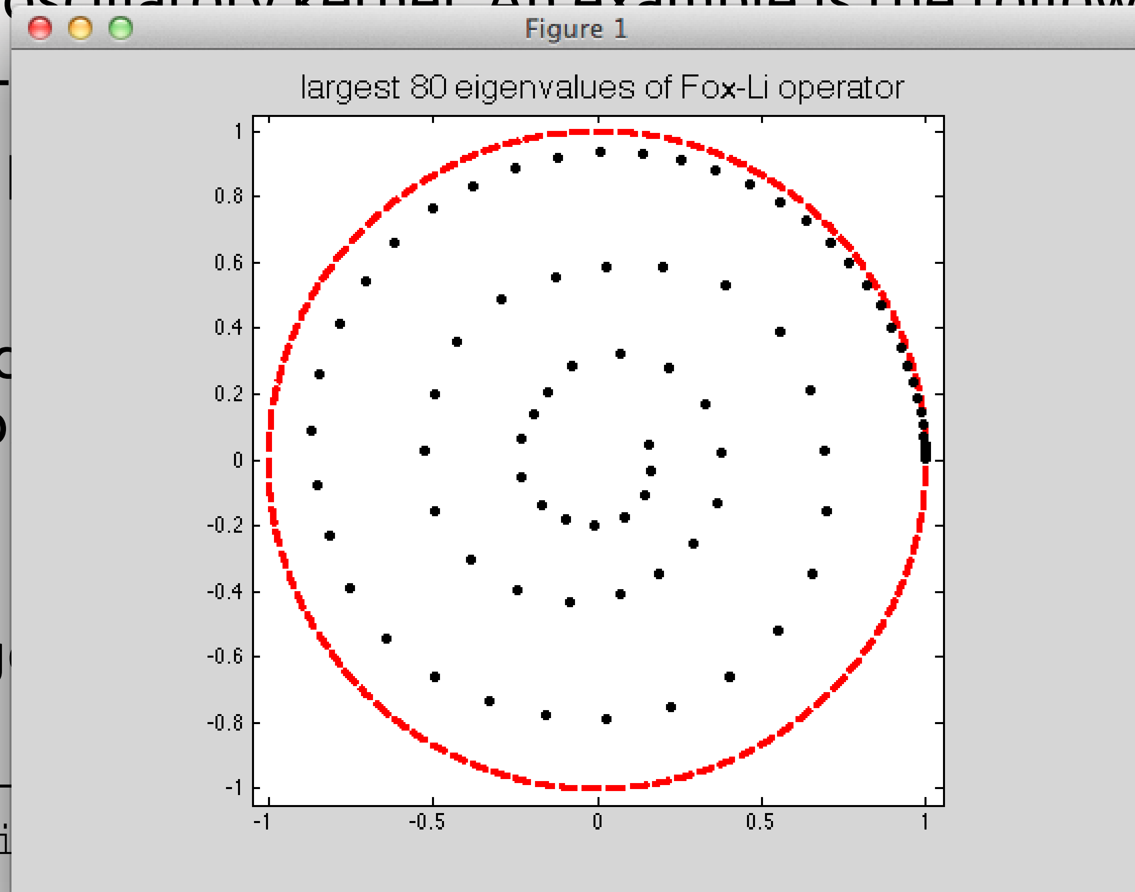


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CHEBFUN EXAMPLES

The quickest way to solve your problem with Chebfun may be to find a similar problem someone else has solved to use as a template. This page connects you to dozens of such templates, called Chebfun Examples. Each example is an M-file producing text and/or graphical output which executes, in most cases, in less than 5 seconds. You can also execute the example with Matlab's PUBLISH command to get a more informative story. Type open(publish('filename')) to see the quickest version on your screen or publish('filename','latex') for a better formatted LaTeX version, which will appear in a directory called html. The published output is also available for direct download as a pdf file.

Each example is signed by the author, and we welcome new contributions. Please send drafts to discuss@chebfun.org with an indication of which section they belong in. To help maintain some uniformity across the examples, please take a look at the formatting conventions.

1. Rootfinding
2. Optimization
3. Quadrature
4. Linear algebra
5. Approximation of functions
6. Complex variables
7. Geometry
8. Statistics
9. Ordinary differential equations
10. Integral and integro-differential equations
11. Partial differential equations

A complete listing of the Examples can be found here.

Please contact us with any questions and comments.

The Future

The Future

+ Improve speed and usability/simplicity

+ Improve ODE and PDE solvers

+ Higher dimensions?

+ Increase developer and user base (incl. publications)

+ Improve connections to real-world applications

+ Port to other languages? (C, Octave, Python?)

The End

The End

Thank you for listening!*

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* and KAUST Award No. KUK-C1-013-04, The EPSRC, and The MathWorks for funding!

Colleague Matrices & Rootfinding

Seek the roots of the Chebyshev polynomial $p_n(x) = \sum_{j=0}^{n} c_j T_j(x)$

Recurrence relation for the Chebyshev polynomials

$$T_0(x) = 1, T_1(x) = x, T_{j+1} = 2xT_j(x) - T_{j-1}$$

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Consider the 'Colleague' matrix

$$\left[\begin{pmatrix} 0 & 1 & 0 & 0 \\ ½ & 0 & ½ & 0 \\ 0 & \ddots & \ddots & \ddots \\ 0 & 0 & ½ & 0 \end{pmatrix} - \frac{1}{2c_n} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_0 & \cdots & c_{n-1} \end{pmatrix}\right]$$

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Consider the 'Colleague' matrix. Eigenvalues are roots of p_n!

$$\left[\begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \ddots & \ddots & \ddots \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} - \frac{1}{2c_n}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_0 & \cdots & c_{n-1} \end{pmatrix}\right]\begin{bmatrix} T_0(r) \\ T_1(r) \\ \vdots \\ T_{n-1}(r) \end{bmatrix} = r\begin{bmatrix} T_0(r) \\ T_1(r) \\ \vdots \\ T_{n-1}(r) \end{bmatrix}$$

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$$½T_{n-1}(r)-\frac{1}{2c_n}\sum_{j=0}^{n} c_j T_j(r)=rT_{n-1}(r) \Rightarrow p_n(0)=0$$