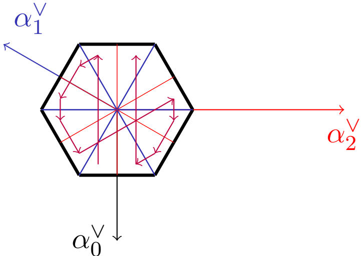
Computer exploration for affine Schubert calculus
Part I: Sage and Sage-Combinat

Jason Bandlow and Nicolas M. Thiéry

With slides from Franco Saliola, Florent Hivert, Dan Drake, William Stein, ...

Laboratoire de Mathématiques d'Orsay, Université Paris Sud
University of California at Davis

July 7, 2010

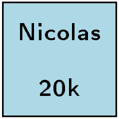


The *-Combinat Project

- *December 2000:* Birth of the project
- *March 2001, SLC 46:*– Call for collaborators for a joint open source software project in algebraic combinatorics.
- *Original platform:* `MuPAD`
- *June 2007-2010:* *-`Combinat` chosen for the NSF FRG "Affine Schubert Calculus"
- *June 2008:* Switch to Sage
- *March 2010, SLC 64:*

It all started there

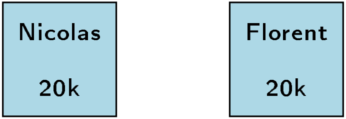
∗-Combinat: 1



Nicolas

20k

*-Combinat: 1+1 =



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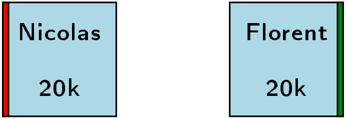

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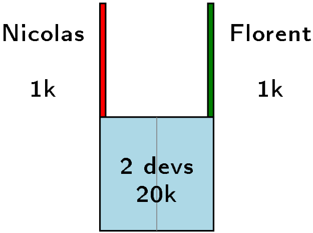

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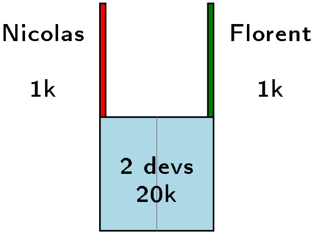

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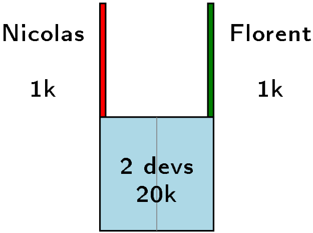

- 95% of development effort are generic
- Opportunity for sharing and mutualisation

∗-Combinat in a nutshell

`http://mupad-combinat.sf.net`
`http://combinat.sagemath.org`

Mission statement: **To improve Sage as an extensible toolbox for computer exploration in combinatorics, and foster code sharing among researchers in this area**

∗-Combinat in a nutshell

- MuPAD: 115k lines of MuPAD, 15k lines of C++, 32k lines of tests, 600 pages of doc
- Sage: 300 tickets / 100k lines integrated in Sage
- Sponsors: ANR, PEPS, NSF, Google Summer of Code

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- A community:

Nicolas Borie, Daniel Bump, **Jason Bandlow**, Adrien Boussicault, Frédéric Chapoton, Vincent Delecroix, Paul-Olivier Dehaye, **Tom Denton**, François Descouens, Dan Drake, Teresa Gomez Diaz, Valentin Feray, Mike Hansen, Ralf Hemmecke, Florent Hivert, **Brant Jones**, Sébastien Labbé, Yann Laigle-Chapuy, Éric Laugerotte, Patrick Lemeur, Andrew Mathas, Xavier Molinero, Gregg Musiker, Jean-Christophe Novelli, Janvier Nzeutchap, **Steven Pon**, **Franco Saliola**, **Anne Schilling**, **Mark Shimozono**, **Lenny Tevlin**, **Nicolas M. Thiéry**, Justin Walker, **Qiang Wang**, Mike Zabrocki, ...
And you ?

Sage's mission

"To create a viable high-quality and open-source alternative to Maple^{TM}, Mathematica^{TM}, Magma^{TM}, and MATLAB^{TM}

...

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and to foster a friendly community of users and developers”

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- *Feb. 2010:* Sage 4.4.2
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- *Current version:* Sage-4.4.4
- 10000 users?
- *Funding* (postdoc, workshops, hardware): NSF, ANR, CNRS, Universities and Institutes, Google, Microsoft Research, . . .

Sage is very young!

Sage has:

- bugs
- inconsistencies
- blank or undocumented areas

Sage does not have:

- native support under Windows
- (working) package under Debian / Ubuntu / ...

Sage is a *distribution* of open source software

ATLAS	Automatically Tuned Linear Algebra Software
BLAS	Basic Fortan 77 linear algebra routines
Bzip2	High-quality data compressor
Cddlib	Double Description Method of Motzkin
Common Lisp	Multi-paradigm and general-purpose programming lang.
CVXOPT	Convex optimization, linear programming, least squares
Cython	C-Extensions for Python
F2c	Converts Fortran 77 to C code
Flint	Fast Library for Number Theory
FpLLL	Euclidian lattice reduction
FreeType	A Free, High-Quality, and Portable Font Engine

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G95	Open source Fortran 95 compiler
GAP	Groups, Algorithms, Programming
GD	Dynamic graphics generation tool
Genus2reduction	Curve data computation
Gfan	Gröbner fans and tropical varieties
Givaro	C++ library for arithmetic and algebra
GMP	GNU Multiple Precision Arithmetic Library
GMP-ECM	Elliptic Curve Method for Integer Factorization
GNU TLS	Secure networking
GSL	Gnu Scientific Library
JsMath	JavaScript implementation of LaTeX

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IML	Integer Matrix Library
IPython	Interactive Python shell
LAPACK	Fortan 77 linear algebra library
Lcalc	L-functions calculator
Libgcrypt	General purpose cryptographic library
Libgpg-error	Common error values for GnuPG components
Linbox	C++ linear algebra library
Matplotlib	Python plotting library
Maxima	computer algebra system
Mercurial	Revision control system
MoinMoin	Wiki

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MPFI	Multiple Precision Floating-point Interval library
MPFR	C library for multiple-precision floating-point computations
ECLib	Cremona's Programs for Elliptic curves
NetworkX	Graph theory
NTL	Number theory C++ library
Numpy	Numerical linear algebra
OpenCDK	Open Crypto Development Kit
PALP	A Package for Analyzing Lattice Polytopes
PARI/GP	Number theory calculator
Pexpect	Pseudo-tty control for Python
PNG	Bitmap image support

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PolyBoRi	Polynomials Over Boolean Rings
PyCrypto	Python Cryptography Toolkit
Python	Interpreted language
Qd	Quad-double/Double-double Computation Package
R	Statistical Computing
Readline	Line-editing
Rpy	Python interface to R
Scipy	Python library for scientific computation
Singular	fast commutative and noncommutative algebra
Scons	Software construction tool
SQLite	Relation database

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Sympow	L-function calculator
Symmetrica	Representation theory
Sympy	Python library for symbolic computation
Tachyon	lightweight 3d ray tracer
Termcap	for writing portable text mode applications
Twisted	Python networking library
Weave	Tools for including C/C++ code within Python
Zlib	Data compression library
ZODB	Object-oriented database

. . . and more!

Sage is a *distribution* of mathematics software

Arbitrary precision arithmetic Algebra	GAP, Maxima, Singular
Algebraic geometry	Singular, Macaulay2 (optionnel)
	GMP, MPFR, MPFI, NTL, . . .
Arithmetic geometry	PARI, NTL, mwrank, ecm, . . .
Symbolic computation	Maxima, Sympy
Exact linear algebra	Linbox, IML
Numerical calculations	GSL, Scipy, Numpy
Combinatorics	Symmetrica, Lrcalc, PALP, Coxe
Graph theory	NetworkX, graphviz (optional)
Group theory	GAP

. . . *and more!*

Sage is a *distribution* of mathematics software

```
> sage -gap
```

```
            #########           ######         ###########           ###
         #############          ######         ############         ####
        ##############         ########        #############       #####
       ###############         ########        #####   ######      #####
      ######         #         #########       #####    #####     ######
     ######                   ##########       #####    #####    #######
     #####                    ##### ####       #####   ######   ########
     ####                    #####  #####      #############   ###  ####
     #####     #######       ####    ####      ###########    ####  ####
     #####     #######      #####    #####     ######        ####   ####
     #####     #######      #####    #####     #####         #############
      #####      #####     ################    #####         #############
      ######     #####     ################    #####         #############
      ################    ##################   #####                ####
       ###############    #####        #####   #####                ####
         #############    #####        #####   #####                ####
          #########      #####          #####  #####                ####

     Information at:  http://www.gap-system.org
     Try '?help' for help. See also  '?copyright' and  '?authors'

   Loading the library. Please be patient, this may take a while.
GAP4, Version: 4.4.10 of 02-Oct-2007, x86_64-unknown-linux-gnu-gcc

gap>
```

Sage is a *distribution* of mathematics software

```
> sage -singular
```

```
                     SINGULAR                             /  Development
 A Computer Algebra System for Polynomial Computations   /   version 3-1-0
                                                       0<
     by: G.-M. Greuel, G. Pfister, H. Schoenemann        \   Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern    \
>
```

Sage is a *distribution* of mathematics software

```
> sage -maxima
```

```
Maxima 5.16.3 http://maxima.sourceforge.net
Using Lisp ECL 9.4.1
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1)
```

Sage is a *distribution* of mathematics software

```
> sage -gp

                  GP/PARI CALCULATOR Version 2.3.3 (released)
          amd64 running linux (x86-64/GMP-4.2.1 kernel) 64-bit version
           compiled: Jul 10 2009, gcc-4.3.2 (Ubuntu 4.3.2-1ubuntu12)
                (readline v5.2 enabled, extended help available)

                     Copyright (C) 2000-2006 The PARI Group

PARI/GP is free software, covered by the GNU General Public License, and
comes WITHOUT ANY WARRANTY WHATSOEVER.

Type ? for help, \q to quit.
Type ?12 for how to get moral (and possibly technical) support.

parisize = 8000000, primelimit = 500000
?
```

Sage is a *distribution* of mathematics software

```
> sage -R

R version 2.6.1 (2007-11-26)
Copyright (C) 2007 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

>
```

Sage *combines* the power of multiple software

Sage includes a large *native library*

- 500k lines of code
- 2500 classes
- 27058 functions
- Rich mathematical content: categories, combinatorics, graphs, number theory, ...
- Many new algorithms

Sage's programming language is *Python*

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- Easy to use as a pocket calculator:

```
python: x = 17
python: x
17
python: 3*x
51
```

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- Easy to read and write:

math: $\left\{17x \;\middle|\; x \in \{0, 1, \ldots, 9\} \text{ and } x \text{ is odd}\right\}$

```
python: [17*x for x in range(10) if x%2 == 1]
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- Easy to learn, with lots of free documentation:
  - Dive into Python (`http://diveintopython.adrahon.org`)
  - Python tutorial(`http://www.ceramiko.ch/python/main.html`)

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- Easy integration of *C/C++/Fortran/...* libraries
- *Cython*: code Python $\longleftrightarrow$ code C.
- Python is becoming a de facto standard for scientific computing

Sage is completely free

"You can read Sylow's Theorem and its proof in Huppert's book in the library . . . then you can use Sylow's Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly . . . .

With this situation two of the most basic rules of conduct in mathematics are violated: In mathematics information is passed on free of charge and everything is laid open for checking."

— J. Neubüser (1993)

(started GAP in 1986)

Freedom of execution

Anywhere, anytime, for any purpose, with only law as limit

Freedom to redistribute copies

or even to *sell* copies

Advantages:

- Technical and administrative simplicity
- Usage by students at home
- Remote computing, large scale calculations
- Sharing of your programs, worksheets

- Non discrimination
- Free access for non academic
- Free access for developing countries

Freedom of study

Advantages:

- Teaching
- Dissemination of science
- Most useful if the code is expressive!
- Reproducibility of scientific results
- Proof checking
- Control over the hypothesis, models and algorithms
- Analysis of bugs and unexpected behavior

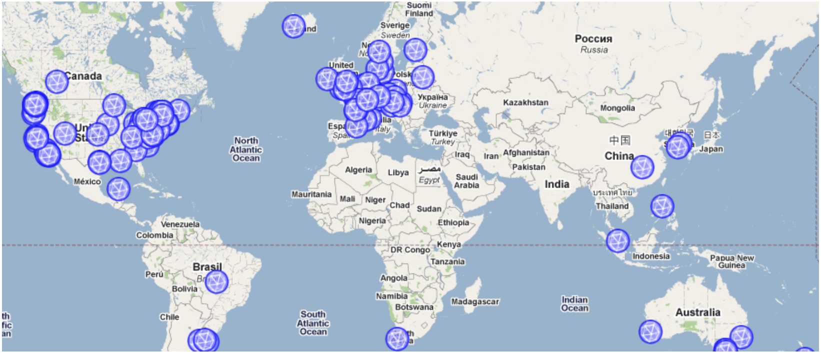
Freedom to improve and publish one's improvements

Advantages:

- Adaptation to local needs (dialect, TODO!)
- Specific developments
- Bug fixes

- Empowering of users
- Mutualisation of efforts
- Importance of communities

Sage's worldwide community



There currently are 184 contributors in 118 different places.

Sage's worldwide community

web sites

`http://www.sagemath.org/`

`http://www.sagemath.fr/`

mailing lists

`sage-devel`: development

`sage-windows`: Windows port

`sage-release`: release management

`sage-algebra`: algebra

`sage-combinat-devel`: combinatorics

`sage-finance` finance

`sage-nt` number theory

`sage-grid` grid computing

`sage-edu` teaching

`sagemath-edu` teaching (in French)

irc-channel

`#sage-devel on freenode.net`

Sage Days!

- Intensive five days workshops (10-120 participants)
  attract new users and developers
  develop new features
- Introductory talks, tutorials
- Plenary conferences from computational experts
- Design brainstorms
- Coding sprints!

Sage Days in 2010

- Sage Days 19: Seattle, WA (January 2010)
- Sage Days 20: Marseille (February 2010)
- Sage Days 20.25: Montreal (March 2010)
- Sage Days 20.5: Fields Institute (May 2010)
- Sage Days 21: Seattle, WA (June 2010)
- Sage-Combinat/Chevie: France (June 2010)
- Sage Days 22: Berkeley, CA (July 2010)
- Sage Days 23: Leiden, Netherlands (July 2010)
- Sage Days 24: Linz, Austria (July 2010)
- Sage Days 25: Mumbai, India (August 2010)
- Sage Days 26: Kaiserslautern, Germany (August 2010)

High-quality code and documentation

All new code is:

- rigorously tested
- well documented
- peer-reviewed

```
> sage -coverage
...
Overall weighted coverage score:  81.1%
Total number of functions:  27058
We need 1061 more function to get to 85% coverage.
We need 2414 more function to get to 90% coverage.
We need 3767 more function to get to 95% coverage.
```

Open source books!

«Calcul Mathématique avec Sage » (July 2010)

Alexandre Casamayou, Guillaume Connan
Thierry Dumont, Laurent Fousse
François Maltey, Matthias Meulien
Marc Mezzarobba, Clément Pernet
Nicolas M. Thiéry, Paul Zimmermann

Development cycle

`http://trac.sagemath.org/`

Tickets (examples: #8154, #8890)

Patchs

New releases every 2–3 weeks

What about combinatorics?

To program or not to program?

A typical computation in algebraic combinatorics involves

- A bit of standard combinatorics
- A bit of standard linear algebra
- A bit of standard group theory
- A bit of standard computer algebra
- A bit of standard ...

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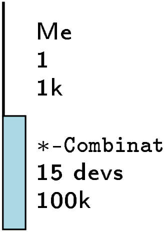
But can I?

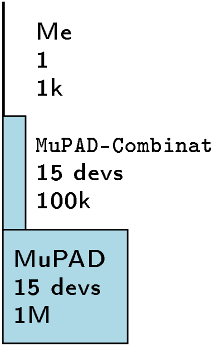
*-Combinat



Me
1
1k

*-Combinat
15 devs
100k

MuPAD-Combinat : limits of the model



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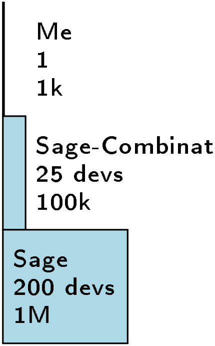
MuPAD
15 devs
1M



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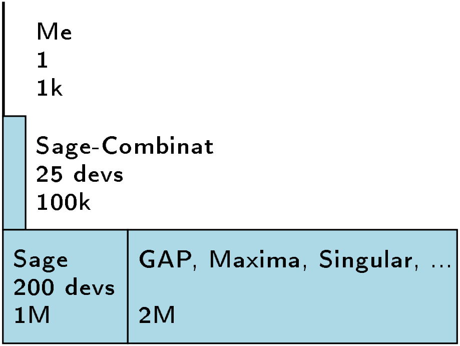
Sage-Combinat
25 devs
100k

Sage
200 devs
1M



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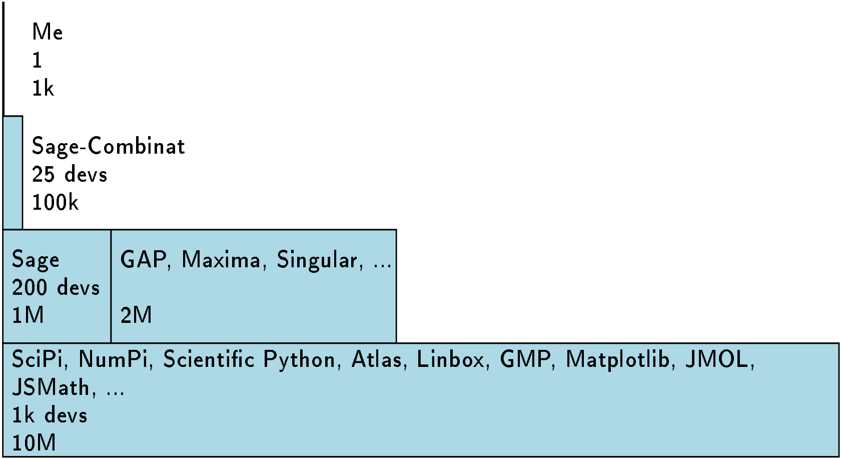
GAP, Maxima, Singular, ...

2M



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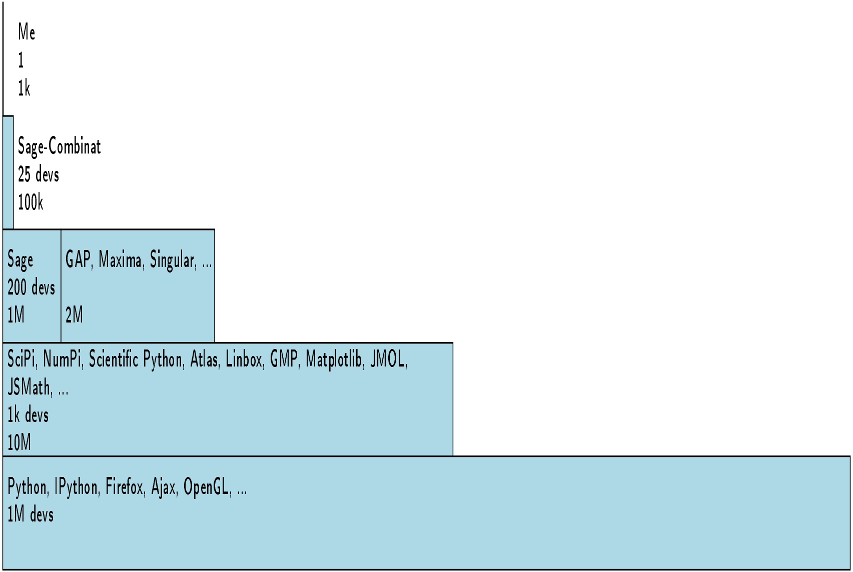
SciPi, NumPi, Scientific Python, Atlas, Linbox, GMP, Matplotlib, JMOL, JSMath, ...
1k devs
10M



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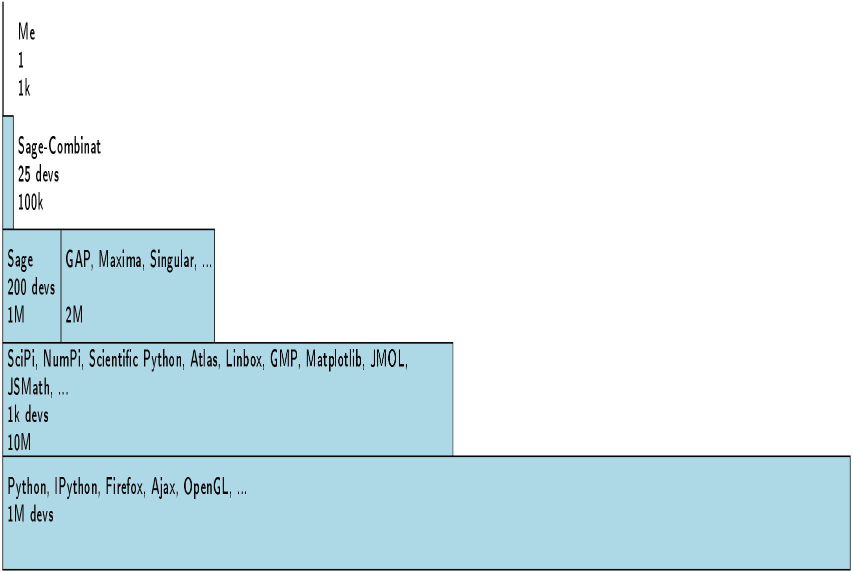
200 devs
1M

GAP, Maxima, Singular, ...
2M

SciPi, NumPi, Scientific Python, Atlas, Linbox, GMP, Matplotlib, JMOL, JSMath, ...
1k devs
10M

Python, IPython, Firefox, Ajax, OpenGL, ...
1M devs

On the shoulders of a giant



Me
1
1k

Sage-Combinat
25 devs
100k

Sage
200 devs
1M

GAP, Maxima, Singular, ...
2M

SciPi, NumPi, Scientific Python, Atlas, Linbox, GMP, Matplotlib, JMOL, JSMath, ...
1k devs
10M

Python, IPython, Firefox, Ajax, OpenGL, ...
1M devs