

Dynamical Systems in Sage

Benjamin Hutz

Department of Mathematics and Statistics
Saint Louis University

November 17, 2019
Sage-days 104

This workshop is supported by NSF grant DMS-1906266

OpenTickets

| Ticket # | Status | Description |
|------------------|--------------|--|
| #28316 | needs-review | is_dynamical_belyi_map() |
| #28292 | needs-review | is_chebyshev and is_lattes |
| #28263 | needs-work | Degree for Affine Maps |
| #28214 | needs-review | nth-preimage tree |
| #28213: | needs-review | all_preperiodic_points() |
| #28212: | needs-review | all_periodic_points() |
| #28173 | needs-review | ls_Newton |
| #28170 | needs-work | use minimal fields |
| #25745 | needs-work | Periodic Proportion Homomorphism over Finite Fields |
| #25701 | needs-work | Implement Sieve algorithm for product_projective space |
| #23816 | needs-review | left action of matrices on scheme points |
| #23806 | needs-info | don't choose default affine patch on projective point init |
| #23720 #23740 | needs-work | Mandelbrot Set for General Polynomials |
| #21129 | needs-review | implementation of Arakelov-Zhang pairing for rational maps |

Other Projects

- Stoll - reduce cluster
- This used to take 10s!

```
sage: R.<t> = PolynomialRing(QQ)
sage: K.<v> = NumberField(t^3 + 16*t^2 - 10496*t + 94208)
sage: PS.<x,y> = ProjectiveSpace(K,1)
sage: f = DynamicalSystem([x^2-29/16*y^2,y^2])
sage: f.rational_preperiodic_graph()
```

- This fails

```
S.<x,y>=CC[]
F = x^3-y^3
F.reduced_form()
```

- Should this work?

```
P.<x,y>=ProjectiveSpace(QQbar,1)
f=DynamicalSystem_projective([x^2+y^2,x*y])
f.sigma_invariants(1)
```

- minimizing field of definition (in all appropriate functions)

```
P.<x,y>=ProjectiveSpace(QQ,1)
f=DynamicalSystem([2*x^3+9*x^2*y-5*x*y^2-12*y^3,-6*y^3])
g=f.normal_form()
```

Wish List

Short term:

- The genus function is broken.
- Improve the speed of dynamical height function.
- Plan for what is needed to implement Berkovich space.
- compute the ramification degree of a critical point
- `is_preperiodic` for finite fields

Long term:

- The genus is function is broken.
- Moduli space portrait for a family of maps with marked points.
(Problem, quasiprojective schemes not implemented.)
 - ▶ Weights to portraits.
 - ▶ Nice to give a portrait and get back a function/functions.
- Berkovich space.
 - ▶ Need to implement \mathbb{C}_p .
 - ▶ How to do type 4 points?
 - ▶ Center and radius.
- Convert between isomorphic finite fields.