<u>Set-up</u> $R = K\Gamma$ $\quad I \triangleleft R$. $\quad \Lambda = R/I$ Finite dimensional

M_Λ rt Λ-mod. <u>Goal</u> Algorithmically construct a proj Λ-resol of M

① Right Λ-mods: $I = \langle \rho \rangle$ $\quad \rho = \{r_1, \dots, r_m\}$ $\quad$ Each $r_i = \sum_j \alpha_{ij} p_{ij}$, $\alpha_{ij} \in K^*$, $p_{ij} \in \mathcal{B} = \{\text{paths}\}$

Assume $\langle \text{arrows} \rangle^N \subseteq I \subseteq \langle \text{arrows} \rangle^2$

$M = (V_v, f_a)_{v \in \Gamma_0, a \in \Gamma_1}$ $\qquad V_v$ fin dim'l v.s./K

if $a: v \to w$ $\quad f_a: V_v \to V_w$ is K-lin

s.t. relations are sat

$\boxed{Ex}$  ...

---

② Projective modules

<u>Prop</u> If Q is a rt proj Λ-mod $Q = \bigoplus v\Lambda$ $\left(\overset{k_i}{\text{basis}}\ v\ \text{Nontip}(I)\right)$

<u>Thm</u> ① If P a rt proj R-mod $P \cong \bigoplus vR$ $\left(\overset{t_i}{\text{basis}}\ v\mathcal{B}\right)$

② If $\hat{P}$ is rt submod of P then $\exists \{f_i\} \subseteq P$ s.t. $\forall i \exists v_i \in \Gamma_0$

with $f_i v_i = f_i$, $\hat{P} = \bigoplus_i f_i R$ and each $f_i R = v_i R$

<u>Step 1</u> Given M_Λ obtain $0 \to \hat{P}^1 \to P^0 \to M \to 0$, an R-presentation of M

$M = (V_v, f_a)$ for each $v \in \Gamma_0$, choose a K-basis $\{m_i^v\}_{i=1}^{d_v}$ of V_v

Then $0 \to \Omega_R^1 M \longrightarrow \bigoplus_v \bigoplus_{i=1}^{d_v} vR \longrightarrow M \to 0$

$\dot{v}_i \longrightarrow m_i^v$

Now if $a: v \to w$ Γ $\quad m_i^v a = \sum_{w'} \sum_j m_j^{w'} c_j^{v}$

Let $f_{i,a}^{*1} = (0, \dots, 0, \underset{\substack{\uparrow \\ v_i \text{ spot}}}{v}, 0, \dots 0)^a - (\dots, c_j^{w'} \dots))$

Then

$\Omega_R^1 M = \bigoplus f_{i,a}^{*1} R$

Step2 Adjust the Λ-presentation of M:

Simplifying notation, we have

$$\underbrace{\bigoplus f_i^{*1} R}_{P^{*1}} \longrightarrow \underbrace{\bigoplus f_i^0 R}_{P^0} \to M \to 0 \qquad f_i^0 \text{ vertices}, \quad f_i^{*1} \in P^0 \text{ st uniform}$$

Break $\{f_i^{*1}\}$ into 2 disj sets $\{f_i^1\}$ and $\{\hat{f}_i^1\}$ where

$$\hat{f}_i^1 \in P^0 I \qquad f_i^1 \notin P^0 I \qquad P^1 = \bigoplus f_i^1 R$$

Then $\underbrace{\bigoplus t(f_i^1)\Lambda}_{P^1/P^1I \,=\, Q^1} \xrightarrow{d^1} \underbrace{\bigoplus f_i^0 \Lambda}_{P^0/P^0I \,=\, Q^0} \to M \to 0$ is a Λ-pres of M

$\left[\; f_i^1 = \sum f_j^0 h_{ji}^{01} \quad d^1 = \left(\overline{h_{ji}^{01}}\right) \right]$

~~Step 2~~ We also have

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & P^0 I \arrow[r, equal] \arrow[d] & P^0 I \arrow[d] & \\
0 \arrow[r] & P^{*1} \arrow[r] \arrow[d] & P^0 \arrow[r] \arrow[d] & M \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & \Omega^1_\Lambda M \arrow[r] & P^0/P^0 I \arrow[r] & M \arrow[r] & 0
\end{tikzcd}$$

Step3 Find $Q^2 \to Q^1 \to Q^0 \to M \to 0$.

$$\begin{tikzcd}
0 \arrow[d] & 0 \arrow[d] \\
P^1 \cap P^0 I \arrow[r] \arrow[d] & P^0 I \arrow[d] \\
P^1 \arrow[r, hook] \arrow[d] & P^{*1} \arrow[d] \\
\Omega^1_\Lambda M \arrow[r, equal] \arrow[d] & \Omega^1_\Lambda M \arrow[d] \\
0 & 0
\end{tikzcd}$$

By Thm, $P^1 \cap P^0 I = \bigoplus f_j^{*2} R$, $f_j^{*2} \in P^1$

If we can find f_j^{*2} we can continue

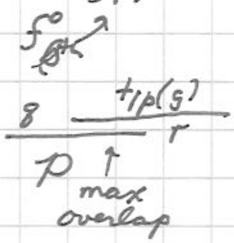
Use right GB theory for $P^0 = \oplus f_i^0 R$

Given $>$ adm order on $\mathcal{B}$, $\mathcal{G}$ a red. GB for I, put $>$ on $\{(0,\dots,0,\text{path},0,\dots,0)\}$ in P^0 "consistent" with $>$.

f_i^0 vertices $\quad f_j^1 \in P^0 \Rightarrow f_j^1 = (h_1,\dots,h_r)$ where $h_j = f_i^0 h_i$, $h_i \in R$

$tip(f_j^1) = (0,\dots,0,p,0,\dots,0)$

Let $g \in G$ (A)



Consider $f_j^1 r - f_\ell^0 g \in \amalg f_i^{1} R$

$= \sum f_i^1 s_i + \sum \hat{f}_j^1 t_j$

$f^2 = f_j^1 r - \sum f_i^1 s_i$

(B)

"minimal"

$f_j^1 r\, t_{ip} g - f_\ell^0 p r g = \sum f_i^1 s_i + \sum \hat{f}_j^1 t_j$

$\hat{f}^2 = f_r^1 t_{ip} g - \sum f_i^1 s_i$

<u>Thm</u> $Q^2 = \oplus f_i^2 R / \oplus f_i^2 I$ & $d^2: Q^2 \to Q^1$ given by $d^2 = (\overline{h_{ji}^{12}})$ where $f_i^2 = \sum f_j^1 h_{ji}^{12}$

then $Q^2 \xrightarrow{d^2} Q^1 \xrightarrow{d^1} Q^0 \to M \to 0$ exact