Affine Stanley symmetric functions

Anne Schilling

Department of Mathematics
University of California at Davis

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Goal

"**Real life example**" for quotient space in Sage that Jason introduced:

- Λ ring of symmetric functions
- Monomial symmetric functions m_λ
- $\Lambda_{(k)} = \Lambda / \langle m_\lambda \mid \lambda_1 > k \rangle$
- dual k-Schur function $\mathfrak{S}_\lambda^{(k)}$ labeled by k-bounded partitions λ form basis for $\Lambda_{(k)}$
- How to access them in Sage?

Outline

1. **Stanley symmetric functions**
   - Definition
   - Properties
2. **Type A affine Stanley symmetric functions**
   - Cyclically decreasing words
   - Affine Stanley symmetric functions
   - Properties
3. **Behind the curtain**
4. **Characters**

Outline

Symmetric group

Definition (Symmetric group)

The symmetric group S_n

- generators $s_1, \ldots, s_{n-1}$
- relations

$$\begin{aligned} s_i s_j &= s_j s_i && \text{for } |i-j| \geq 2 \\ s_i s_{i+1} s_i &= s_{i+1} s_i s_{i+1} \\ s_i^2 &= 1 \end{aligned}$$

Stanley symmetric functions

Introduced in 1984 by Stanley

- used to study # of reduced words of $w \in S_n$
- closely related to Schubert polynomials of Lascoux and Schützenberger (related to geometry of flag varieties)

nilCoxeter algebra

Definition (nilCoxeter algebra)

The nilCoxeter algebra

- generators $u_1, \ldots, u_{n-1}$
- relations

$$\begin{aligned} u_i u_j &= u_j u_i && \text{for } |i-j| \geq 2 \\ u_i u_{i+1} u_i &= u_{i+1} u_i u_{i+1} \\ u_i^2 &= 0 \end{aligned}$$

$\mathbb{C}[S_n]$ group algebra of symmetric group

inner product $\langle w, v \rangle = \delta_{w,v}$

linear operators $u_i : \mathbb{C}[S_n] \to \mathbb{C}[S_n]$ for $1 \leq i < n$

$$u_i w = \begin{cases} s_i w & \text{if } \ell(s_i w) > \ell(w) \\ 0 & \text{else} \end{cases}$$

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Definition

...by Fomin-Stanley using the nilCoxeter algebra

Definition

$$F_w(x) = \sum_{a=(a_1,\ldots,a_\ell)} \langle A_{a_\ell}(u)\cdots A_{a_1}(u)\cdot 1, w\rangle x_1^{a_1}\cdots x_\ell^{a_\ell}$$

where a is a composition and

$$A_k(u) = \sum_{b_1>\cdots>b_k} u_{b_1}\cdots u_{b_k}$$

- symmetry follows since $A_k(u)$ commute
- Stanley symmetric functions are stable limits of Schubert polynomials

$$F_w = \lim_{s\to\infty} \mathfrak{S}_{1^s\times w}$$

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Properties

Theorem (Stanley)

1. $F_w(x)$ *is a symmetric function.*
2. $[x_1 \cdots x_{\ell(w)}]F_w(x) =$ *number of reduced words for* w
3. ***Unique dominant term*** *in monomial expansion:*

$$F_w = m_{\mu(w)} + \sum_{\lambda < \mu(w)} a_{w\lambda} m_\lambda$$

4. ***Conjugacy formula:*** $\omega(F_w) = F_{w^*}$ *where* $* : w_1 \cdots w_n \to (n+1-w_n) \cdots (n+1-w_1)$

Theorem (Edelman-Greene, Lascoux-Schützenberger)

The coefficients $a_{w\lambda}$ *in the Schur expansion* $F_w = \sum_\lambda a_{w\lambda} s_\lambda$ *are nonnegative.*

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Reference

Type A affine Stanley symmetric functions:

- Thomas Lam
  Affine Stanley symmetric functions
  J. Amer. Math. Soc. 21 (2008), no. 1, 259–281

Type C affine Stanley symmetric functions

- Thomas Lam, Anne Schilling, Mark Shimozono
  Schubert Polynomials for the affine Grassmannian of the symplectic group
  Mathematische Zeitschrift 264(4) (2010) 765-811

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Affine symmetric group

Definition

The affine symmetric group $\tilde{S}_n$

- generators $s_0, s_1, \ldots, s_{n-1}$
- relations

$$
\begin{aligned}
s_i s_j &= s_j s_i && \text{for } |i-j| \geq 2 \\
s_i s_{i+1} s_i &= s_{i+1} s_i s_{i+1} \\
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Remark

All indices $i \in [0, n-1]$ are taken modulo n.

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Representation of U_n on $\mathbb{C}[\tilde{S}_n]$

$$u_i w = \begin{cases} s_i w & \text{if } \ell(s_i w) > \ell(w) \\ 0 & \text{else} \end{cases}$$

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Cyclically decreasing words

Definition

Let $a = a_1 a_2 \ldots a_k$ be a word without repetition, $a_i \in [0, n-1]$.
$A = \{a_1, \ldots, a_k\} \subset [0, n-1]$.
a is cyclically decreasing if for all i such that $i, i+1 \in A$, $i+1$ preceeds i in a.

Example

$n = 9$
The word 082654 is cyclically decreasing.

Definition

$u \in U_n$ is cyclically decreasing if $u = u_a = u_{a_1} \cdots u_{a_k}$ for some cyclically decreasing word a.

u is completely determined by $A \Rightarrow$ write u_A

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$$\tilde{F}_w(x) = \sum_{a=(a_1,\ldots,a_\ell)} \langle h_{a_\ell}(u) \cdots h_{a_1}(u) \cdot 1, w \rangle x_1^{a_1} \cdots x_\ell^{a_\ell}$$

where

$$h_k(u) = \sum_{A \in \binom{[0,n-1]}{k}} u_A$$

Subspaces of Λ

Λ ring of symmetric functions
$\mathcal{P}^k$ set of partitions $\{\lambda \mid \lambda_1 \le k\}$ $\qquad k = n-1$

$$\Lambda_{(k)} := \mathbb{C}\langle h_\lambda \mid \lambda \in \mathcal{P}^k\rangle = \mathbb{C}\langle e_\lambda \mid \lambda \in \mathcal{P}^k\rangle = \mathbb{C}\langle p_\lambda \mid \lambda \in \mathcal{P}^k\rangle$$
$$\Lambda^{(k)} := \mathbb{C}\langle m_\lambda \mid \lambda \in \mathcal{P}^k\rangle$$

Hall inner product $\langle \cdot, \cdot \rangle$:
for $f \in \Lambda_{(k)}$ and $g \in \Lambda^{(k)}$ define $\langle f, g\rangle$ as the usual Hall inner product in Λ
$\{h_\lambda\}$ and $\{m_\lambda\}$ with $\lambda \in \mathcal{P}^k$ form dual bases of $\Lambda_{(k)}$ and $\Lambda^{(k)}$

$\Lambda_{(k)}$	is a subalgebra
$\Lambda^{(k)}$	is **not** closed under multiplication, but comultiplication

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Properties

Theorem

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2. $[x_1 \cdots x_{\ell(w)}]\tilde{F}_w(x) =$ *number of reduced words for* w
3. ***Unique dominant term*** *in monomial expansion:*

$$\tilde{F}_w = m_{\mu(w)} + \sum_{\lambda < \mu(w)} b_{w\lambda} m_\lambda$$

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Grassmannian elements

Definition

$w \in \tilde{S}_n$ is Grassmannian if it is a minimal coset representative of $\tilde{S}_n/S_n$ (i.e. all reduced words end in s_0).

Theorem

$\{\tilde{F}_w \mid w \in \tilde{S}_n/S_n\}$ form a basis of $\Lambda^{(k)}$ for $k = n-1$.

$\tilde{F}_w$ indexed by Grassmannians are the dual k-Schur functions of Lapointe-Morse $\mathfrak{S}_\lambda^{(k)} \in \Lambda^{(k)}$.

Definition

k-Schur functions are the dual basis in $\Lambda_{(k)}$ of $\{\mathfrak{S}_\lambda^{(k)} \mid \lambda \in \mathcal{P}^k\}$.

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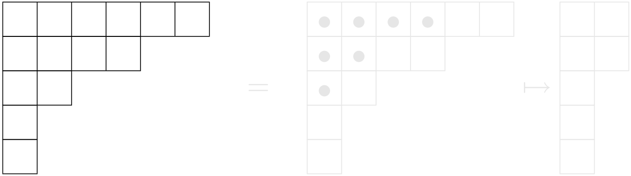
Dual k-Schur functions

Bijection $\tilde{S}_n / S_n \to \mathcal{P}^k$

$k = 2$



$\{\mathfrak{S}^{(k)}_\lambda \mid \lambda \in \mathcal{P}^k\}$ basis of $\Lambda^{(k)} = \mathbb{C}\langle m_\lambda \mid \lambda \in \mathcal{P}^k\rangle$

$\{s^{(k)}_\lambda \mid \lambda \in \mathcal{P}^k\}$ basis of $\Lambda_{(k)} = \mathbb{C}\langle h_\lambda \mid \lambda \in \mathcal{P}^k\rangle$

$\langle s^{(k)}_\mu, \mathfrak{S}^{(k)}_\lambda\rangle = \delta_{\lambda\mu}$ dual bases

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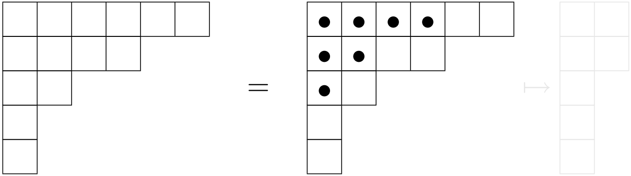

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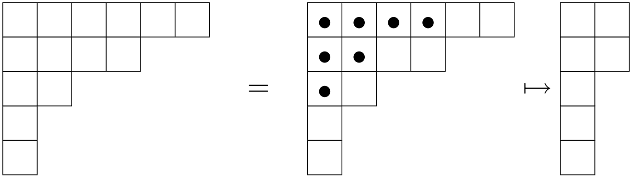

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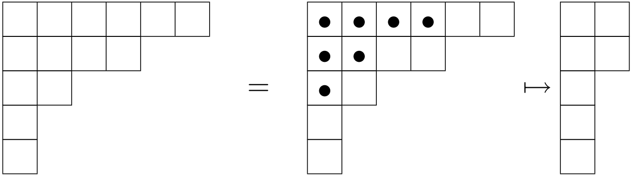
$k = 2$ 

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Reference

- Jason Bandlow, Anne Schilling, Mike Zabrocki
  The Murnaghan-Nakayama rule for k-Schur functions
  preprint arXiv:1004.4886

Characters

k-characters:

$$p_\nu = \sum_{\lambda \in \mathcal{P}^k} \chi^{(k)}_{\lambda,\nu} \, s^{(k)}_\lambda$$

$$\mathfrak{S}^{(k)}_\nu = \sum_{\lambda \in \mathcal{P}^k} \frac{1}{z_\lambda} \chi^{(k)}_{\nu,\lambda} \, p_\lambda$$

Dual version:

$$p_\nu = \sum_{\lambda \in \mathcal{P}^k} \tilde{\chi}^{(k)}_{\lambda,\nu} \, \mathfrak{S}^{(k)}_\lambda$$

$$s^{(k)}_\nu = \sum_{\lambda \in \mathcal{P}^k} \frac{1}{z_\lambda} \tilde{\chi}^{(k)}_{\nu,\lambda} \, p_\lambda$$