

Parity Predictions

Arijit Sehanobish

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As I have discussed in my presentation I am looking for lawful curves over $Q(i)$ and $Q(\sqrt{-2})$. So far I have not found a single example so far. So what do I mean by having found no example since during my presentation I gave two examples namely $1369e1$ over $Q(\sqrt[4]{-37})$ and $49a1$ over $Q(i)$. My aim is to find curves over imaginary quadratic fields which cannot be defined over Q . We shall call those examples as honest examples. From now on we will be considering only honest examples. In [2], one of the equivalent criteria for lawful curves is the following:

If E/K is an elliptic curve then K should not have any real places and E should have good reduction over an abelian extension of K .

In [1] and [3] Connell and Rohrlich classify elliptic curves over Q which has good reduction over an abelian extension of Q . In fact Rohrlich constructs an infinite family of such curves. So the question relevant to our situation is to classify elliptic curves over a imaginary quadratic field (or more generally over a totally imaginary field) which has good reduction over an abelian extension of the base field. Can one construct infinite families of such curves? But looking at the numerical evidence or rather lack of it, it seems that these curves are a little hard to find.

Meanwhile I will continue my search to find these curves.

References

- [1] I. Connell, Good reduction of elliptic curves in abelian extensions, J. Reine Angew. Math.436(1993), 155 – 175
- [2] T. Dokchitser, V. Dokchitser, Elliptic curves with all quadratic twists of positive rank, Acta Arith. 137 (2009), 193 – 197
- [3] D. Rohrlich, Non-vanishing of L -functions and structure of Mordell-Weil groups, J. Reine Angew. Math.417 (1991), 1 – 25