The Sage system

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With slides from Franco Saliola, Florent Hivert, Dan Drake, William Stein, ...

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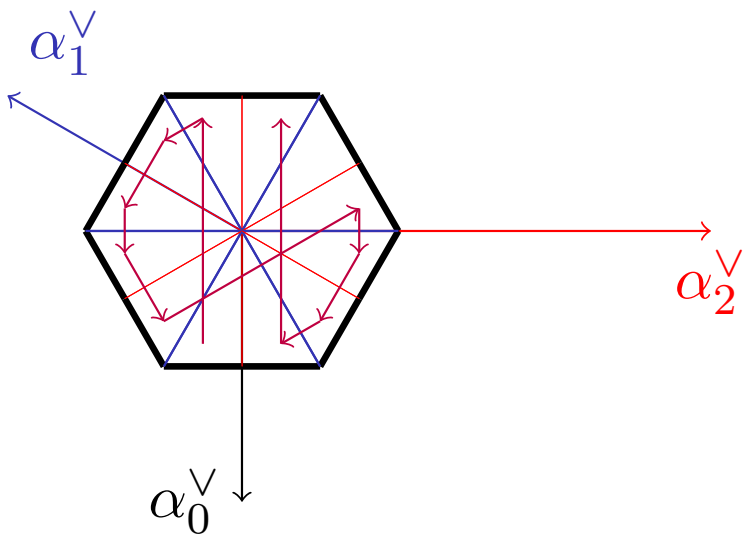
Sage Days 30, Acadia University, 02/05/2011



Outline

Sage?

Sage is a *distribution* of open source software

Software included in Sage:

ATLAS	Automatically Tuned Linear Algebra Software
BLAS	Basic Fortan 77 linear algebra routines
Bzip2	High-quality data compressor
Cddlib	Double Description Method of Motzkin
Common Lisp	Multi-paradigm and general-purpose programming lang.
CVXOPT	Convex optimization, linear programming, least squares
Cython	C-Extensions for Python
F2c	Converts Fortran 77 to C code
Flint	Fast Library for Number Theory
FpLLL	Euclidian lattice reduction
FreeType	A Free, High-Quality, and Portable Font Engine

Sage is a *distribution* of open source software

Software included in Sage:

G95	Open source Fortran 95 compiler
GAP	Groups, Algorithms, Programming
GD	Dynamic graphics generation tool
Genus2reduction	Curve data computation
Gfan	Gröbner fans and tropical varieties
Givaro	C++ library for arithmetic and algebra
GMP	GNU Multiple Precision Arithmetic Library
GMP-ECM	Elliptic Curve Method for Integer Factorization
GNU TLS	Secure networking
GSL	Gnu Scientific Library
JsMath	JavaScript implementation of LaTeX

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Software included in Sage:

IML	Integer Matrix Library
IPython	Interactive Python shell
LAPACK	Fortan 77 linear algebra library
Lcalc	L-functions calculator
Libgcrypt	General purpose cryptographic library
Libgpg-error	Common error values for GnuPG components
Linbox	C++ linear algebra library
Matplotlib	Python plotting library
Maxima	computer algebra system
Mercurial	Revision control system
MoinMoin	Wiki

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Software included in Sage:

MPFI	Multiple Precision Floating-point Interval library
MPFR	C library for multiple-precision floating-point computations
ECLib	Cremona's Programs for Elliptic curves
NetworkX	Graph theory
NTL	Number theory C++ library
Numpy	Numerical linear algebra
OpenCDK	Open Crypto Development Kit
PALP	A Package for Analyzing Lattice Polytopes
PARI/GP	Number theory calculator
Pexpect	Pseudo-tty control for Python
PNG	Bitmap image support

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Software included in Sage:

PolyBoRi	Polynomials Over Boolean Rings
PyCrypto	Python Cryptography Toolkit
Python	Interpreted language
Qd	Quad-double/Double-double Computation Package
R	Statistical Computing
Readline	Line-editing
Rpy	Python interface to R
Scipy	Python library for scientific computation
Singular	fast commutative and noncommutative algebra
Scons	Software construction tool
SQLite	Relation database

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Software included in Sage:

Sympow	L-function calculator
Symmetrica	Representation theory
Sympy	Python library for symbolic computation
Tachyon	lightweight 3d ray tracer
Termcap	for writing portable text mode applications
Twisted	Python networking library
Weave	Tools for including C/C++ code within Python
Zlib	Data compression library
ZODB	Object-oriented database

. . . and more!

Sage is a *distribution* of mathematics software

Arbitrary precision arith.	GMP, MPFR, MPFI, NTL, ...
Algebra	GAP, Maxima, Singular
Algebraic geometry	Singular, Macaulay2 (optional)
Arithmetic geometry	PARI, NTL, mwrank, ecm, ...
Symbolic computation	Maxima, Sympy
Exact linear algebra	Linbox, IML
Numerical calculations	GSL, Scipy, Numpy
Combinatorics	Symmetrica, Lrcalc, PALP, Coxeter 3, Chevie
Graph theory	NetworkX, graphviz (optional)
Group theory	GAP

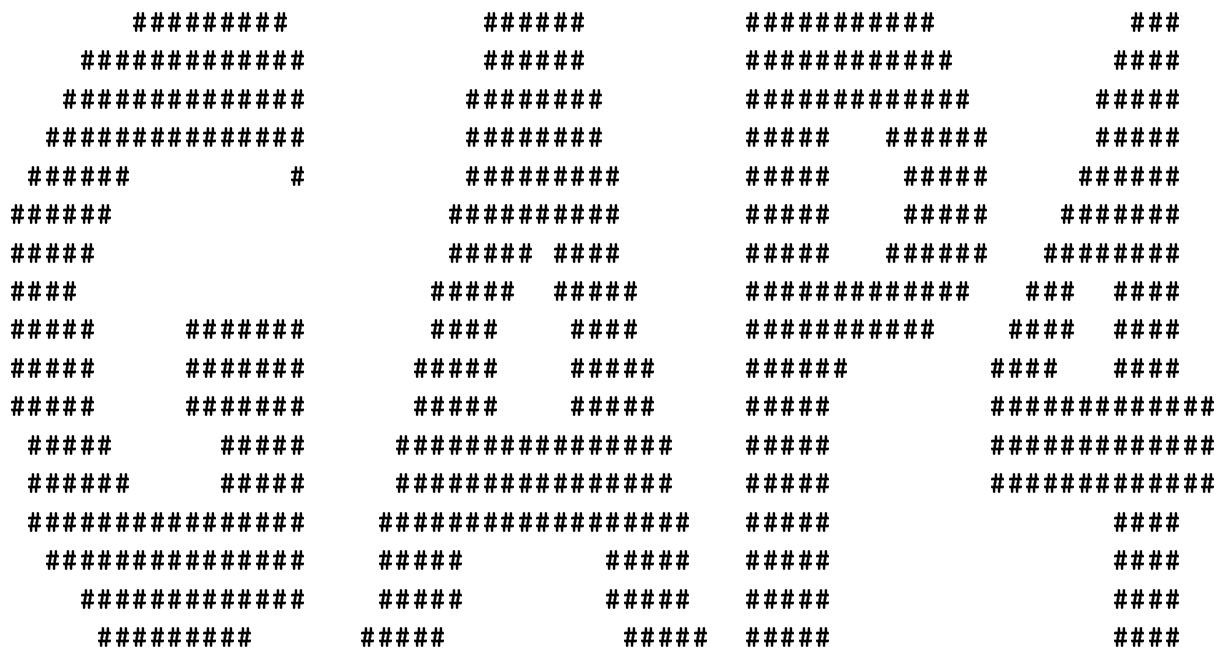
... and more!

Sage Demo

Demo!

Sage is a *distribution* of mathematics software

```
> sage -gap
```



```
    Information at:  http://www.gap-system.org
    Try '?help' for help. See also  '?copyright' and  '?authors'

   Loading the library. Please be patient, this may take a while.
GAP4, Version: 4.4.12 of 17-Dec-2008, x86_64-unknown-linux-gnu-gcc

gap>
```

Sage is a *distribution* of mathematics software

```
> sage -singular

                     SINGULAR                                 /  Development
 A Computer Algebra System for Polynomial Computations      /   version 3-1-1
                                                          0<
     by: G.-M. Greuel, G. Pfister, H. Schoenemann           \   Feb 2010
FB Mathematik der Universitaet, D-67653 Kaiserslautern       \
>
```

Sage is a *distribution* of mathematics software

```
> sage -maxima

Maxima 5.22.1 http://maxima.sourceforge.net
using Lisp ECL 10.4.1
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1)
```

Sage is a *distribution* of mathematics software

```
> sage -gp

        GP/PARI CALCULATOR Version 2.4.3 (development svn-12623)
      amd64 running linux (x86-64/GMP-4.2.1 kernel) 64-bit version
       compiled: Apr 20 2011, gcc-4.4.3 (Ubuntu 4.4.3-4ubuntu5)
            (readline v6.1 enabled, extended help enabled)

                  Copyright (C) 2000-2008 The PARI Group

PARI/GP is free software, covered by the GNU General Public License, and
comes WITHOUT ANY WARRANTY WHATSOEVER.

Type ? for help, \q to quit.
Type ?12 for how to get moral (and possibly technical) support.

parisize = 8000000, primelimit = 500509
?
```

Sage is a *distribution* of mathematics software

```
> sage -R

R version 2.10.1 (2009-12-14)
Copyright (C) 2009 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

>
```

Sage *combines* the power of multiple software

Demo!

Sage includes a large *native library*

Sage is also *new code*, providing
new or improved functionality
not previously available.

- 500k lines of code
- 2500 classes
- 27200 functions
- Rich mathematical content: categories, combinatorics, graphs, number theory, ...
- Many new algorithms

Sage's mission

"To create a viable high-quality and open-source alternative to Maple^{TM}, Mathematica^{TM}, Magma^{TM}, and MATLAB^{TM}

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"To create a viable high-quality and open-source alternative to Maple™, Mathematica™, Magma™, and MATLAB™

...

and to foster a friendly community of users and developers"

A short history of Sage

- *2002:* Open Source Computer Algebra workshop in Lyon
- *1999-2005:* William Stein writes over 25,000 lines of Magma code for his research, and realizes that Magma was a bad long term investment since he couldn't see or modify the internals
- *Feb. 2005:* Sage 0.1, a Python library linking together PARI, Maxima, Python, Singular, GAP.

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- *Feb. 2010:* Sage 4.4.2
  Sage Days 20, Luminy (France), 120 participants
- *Jan. 2011:* Sage 4.6.1
- 10000 users?
- *Funding* (postdoc, workshops, hardware): NSF, ANR, CNRS, Universities and Institutes, Google, Microsoft Research, ...

Sage is very young!

Sage has:

- bugs
- inconsistencies
- blank or undocumented areas

Sage lacks:

- native support under Windows (upcoming)
- (working) packages under Debian / Ubuntu / ...
- Proper modularization

Python

Sage's programming language is *Python*

Python is an interpreted multiparadigm programming language

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- Easy to use as a pocket calculator:

```
python: x = 17
python: x
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- Easy to read and write:

math: $\left\{17x \;\middle|\; x \in \{0, 1, \ldots, 9\} \text{ and } x \text{ is odd}\right\}$

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- Easy to learn, with lots of free documentation:
  - Dive into Python (`http://diveintopython.adrahon.org`)
  - Python tutorial (`http://www.ceramiko.ch/python/main.html`)

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- *Cython*:
- Python is becoming a major platform for scientific computing

Freedom!

Sage is completely free

"You can read Sylow's Theorem and its proof in Huppert's book in the library . . . then you can use Sylow's Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly . . . .

With this situation two of the most basic rules of conduct in mathematics are violated: In mathematics information is passed on free of charge and everything is laid open for checking."

— *J. Neubüser (1993)*

(started GAP in 1986)

Freedom of execution

Anywhere, anytime, for any purpose, with only law as limit

Freedom to redistribute copies

or even to *sell* copies

Advantages:

- Technical and administrative simplicity
- Usage by students at home
- Remote computing, large scale calculations
- Sharing of your programs, worksheets

- Non discrimination
- Free access for non academic
- Free access for developing countries

Freedom of study

Advantages:

- Teaching
- Dissemination of science
- Most useful if the code is *expressive*!
- Reproducibility of scientific results
- Proof checking
- Control over the hypothesis, models, and algorithms
- Analysis of bugs and unexpected behavior

Freedom to improve and publish one's improvements

Advantages:

- Adaptation to local needs (dialects, conventions)
- Specific developments
- Bug fixes

- Empowering of users
- Mutualisation of efforts
- Importance of being in a community

Community

Sage's worldwide community

There currently are **235** contributors in **159** different places.

Sage's worldwide community

web sites

`http://www.sagemath.org/`

`http://www.sagemath.fr/`

mailing lists

`sage-devel`: development

`sage-windows`: Windows port

`sage-release`: release management

`sage-algebra`: algebra

`sage-combinat-devel`: combinatorics

`sage-finance`: finance

`sage-nt`: number theory

`sage-grid`: grid computing

`sage-edu`: teaching

`sagemath-edu`: teaching (in French)

irc-channel

`#sagemath` on `freenode.net`

Sage Days in 2010

- Sage Days 19: Seattle, WA (January 2010)
- Sage Days 20: Marseille (February 2010)
- Sage Days 20.25: Montreal (March 2010)
- Sage Days 20.5: Fields Institute (May 2010)
- Sage Days 21: Seattle, WA (June 2010)
- Sage-Combinat/Chevie Workshop: France (June 2010)
- Sage Days 22: Berkeley, CA (July 2010)
- Sage Days 23: Leiden, Netherlands (July 2010)
- Sage Days 23.5: Kaiserslautern, Germany (July 2010)
- Sage Days 24: Linz, Austria (July 2010)
- Sage Days 25: Mumbai, India (August 2010)
- Sage Days 25.5 Montréal, Canada (September 2010)
- Sage Days 26 Seattle, Washington (December 7-10, 2010)

Sage Days in 2011

- Joint Math Meetings: New Orleans, LA (January 2011)
- Sage Days 27: Seattle, WA (January 2011)
- Sage Days 28: Orsay, France (January 2011)
- Sage Days 29: Seattle, WA (March 2011)
- $\rightarrow$ Sage Days 30: Wolfville, NS (May 2011)
- Sage Days 31: Seattle, WA (June 2011)
- Sage Education Days 3: Seattle, WA (June 2011)
- Sage Days X, for some $X > 31$: South Korea (Oct 2011)

Some open source books!

- *Calcul Mathématique avec Sage*

  Alexandre Casamayou, Guillaume Connan
  Thierry Dumont, Laurent Fousse
  François Maltey, Matthias Meulien
  Marc Mezzarobba, Clément Pernet
  Nicolas M. Thiéry, Paul Zimmermann

  http://sagebook.gforge.inria.fr/
  July 2010: 1.0 online August 2011: printed

  Follow up to: *Calcul formel, mode d'emploi*
  Dumas, Gomez, Salvy, Zimmermann

- *The Sage tutorial:* http://www.sagemath.org/doc
  David Joyner, William Stein et al.

- *A First Course in Linear Algebra:* http://linear.ups.edu
  Robert Beezer

Development model

Sage's design principles

- Developed by a community of users, for users
- Open source from the ground up (GPL)

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- Based on a standard programming language (Python)
- Bazaar development model
- Active proselytism

High-quality code and documentation

All new code is:

- rigorously tested
- well documented
- peer-reviewed

```
> sage -coverageall
...
Overall weighted coverage score:  84.8%
Total number of functions:  27200
We need   47 more function to get to 85% coverage.
We need 1407 more function to get to 90% coverage.
We need 2767 more function to get to 95% coverage.
```

Development cycle

`http://trac.sagemath.org/`

Tickets (examples: #8154, #8890)

Patchs

New releases roughly every other month

To program or not to program?

A typical computation in algebraic combinatorics involves

- A bit of standard combinatorics
- A bit of standard linear algebra
- A bit of standard group theory
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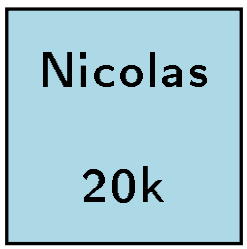
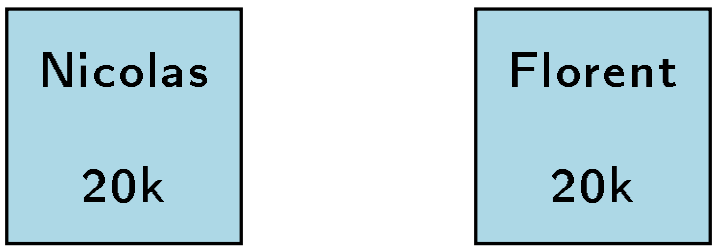
But can I?

*-Combinat: it all started there

*-Combinat: 1



*-Combinat: 1+1 =



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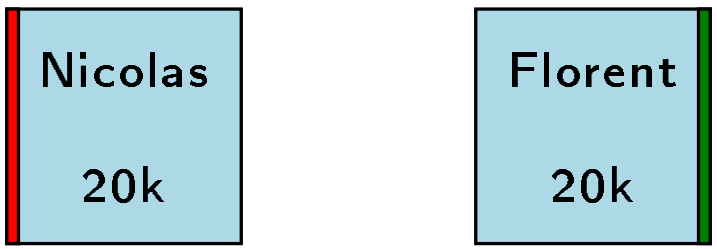
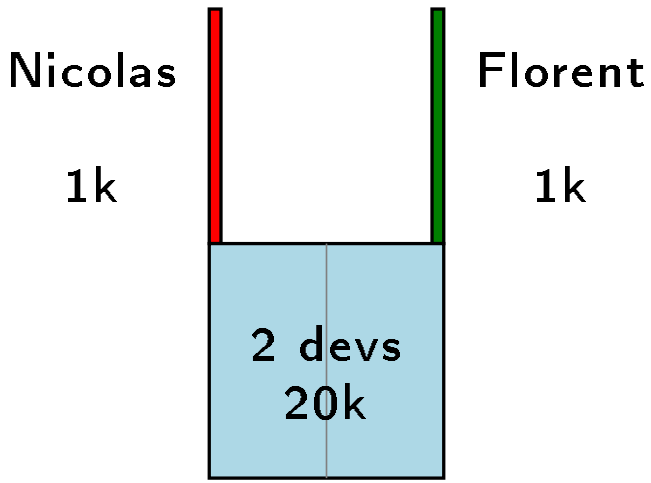


∗-Combinat: 1+1 = 1.1



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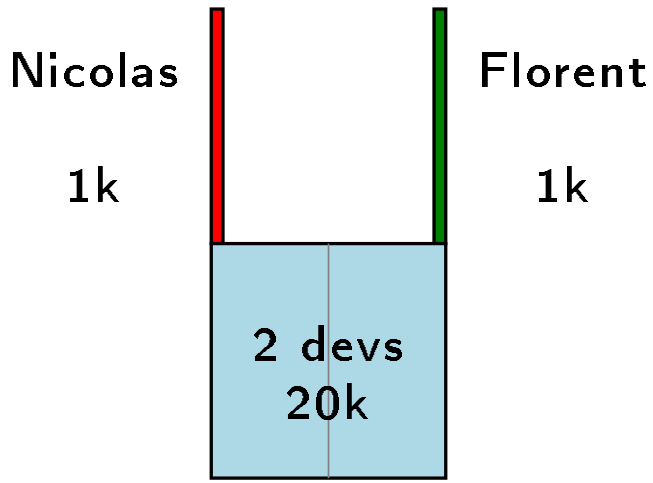

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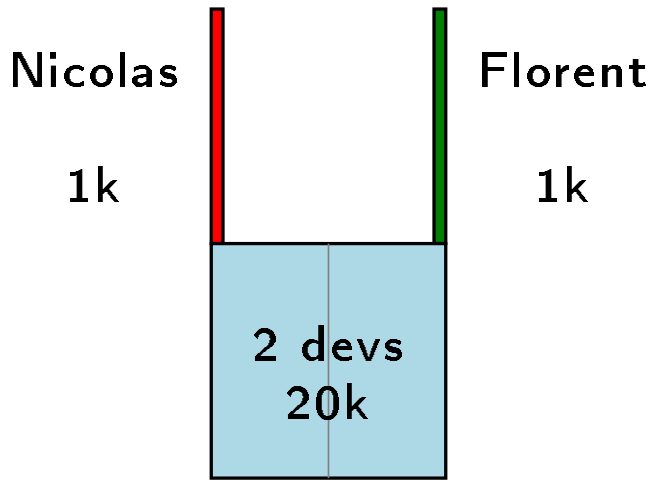

- 95% of development effort are generic
- Opportunity for sharing and mutualisation

*-Combinat in a nutshell

`http://mupad-combinat.sf.net`
`http://combinat.sagemath.org`

Mission statement:

"To improve MuPAD/Sage as an extensible toolbox for computer exploration in combinatorics, and foster code sharing among researchers in this area"

*-Combinat in a nutshell

- MuPAD: 115k lines of MuPAD, 15k lines of C++, 32k lines of tests, 600 pages of doc
- Sage: 300 tickets / 100k lines integrated in Sage
- Sponsors: ANR, PEPS, NSF, Google Summer of Code

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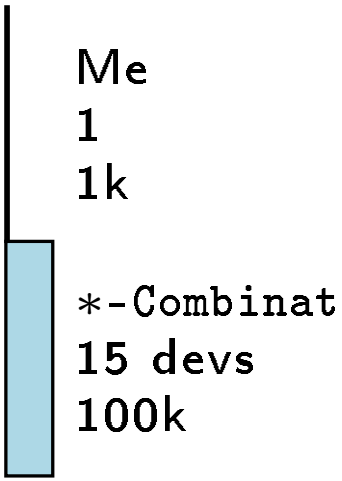
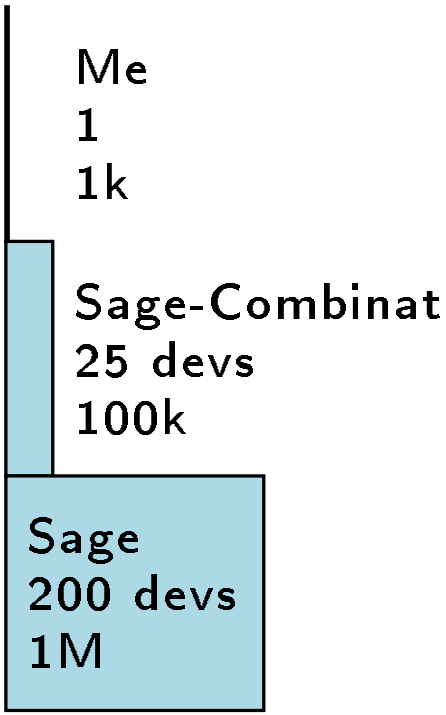
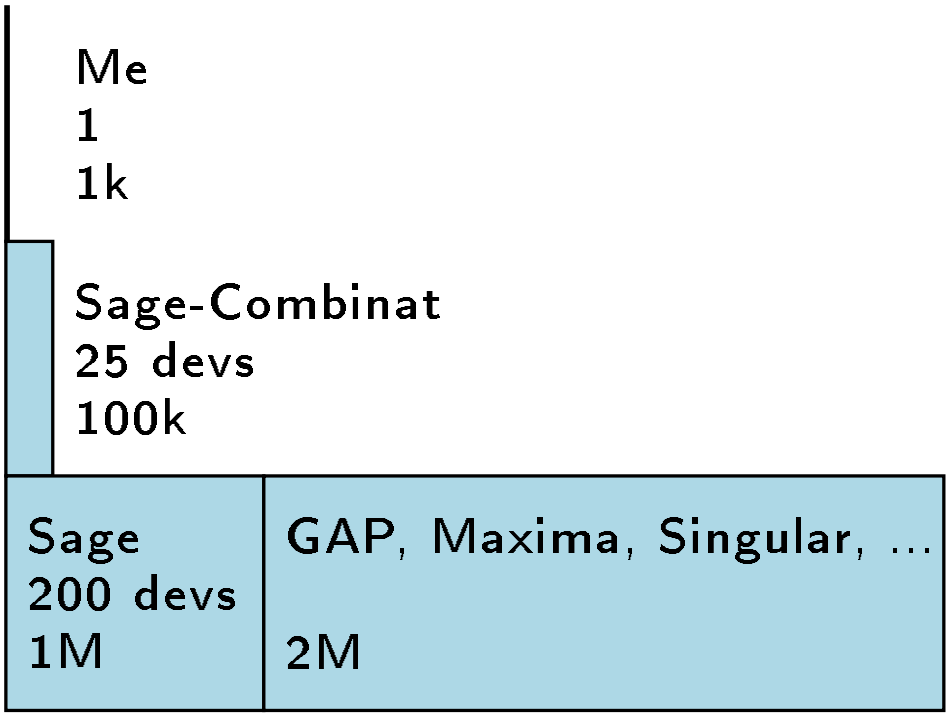
And you ?

*-Combinat




Me
1
1k
Sage-Combinat
25 devs
100k
Sage
200 devs
1M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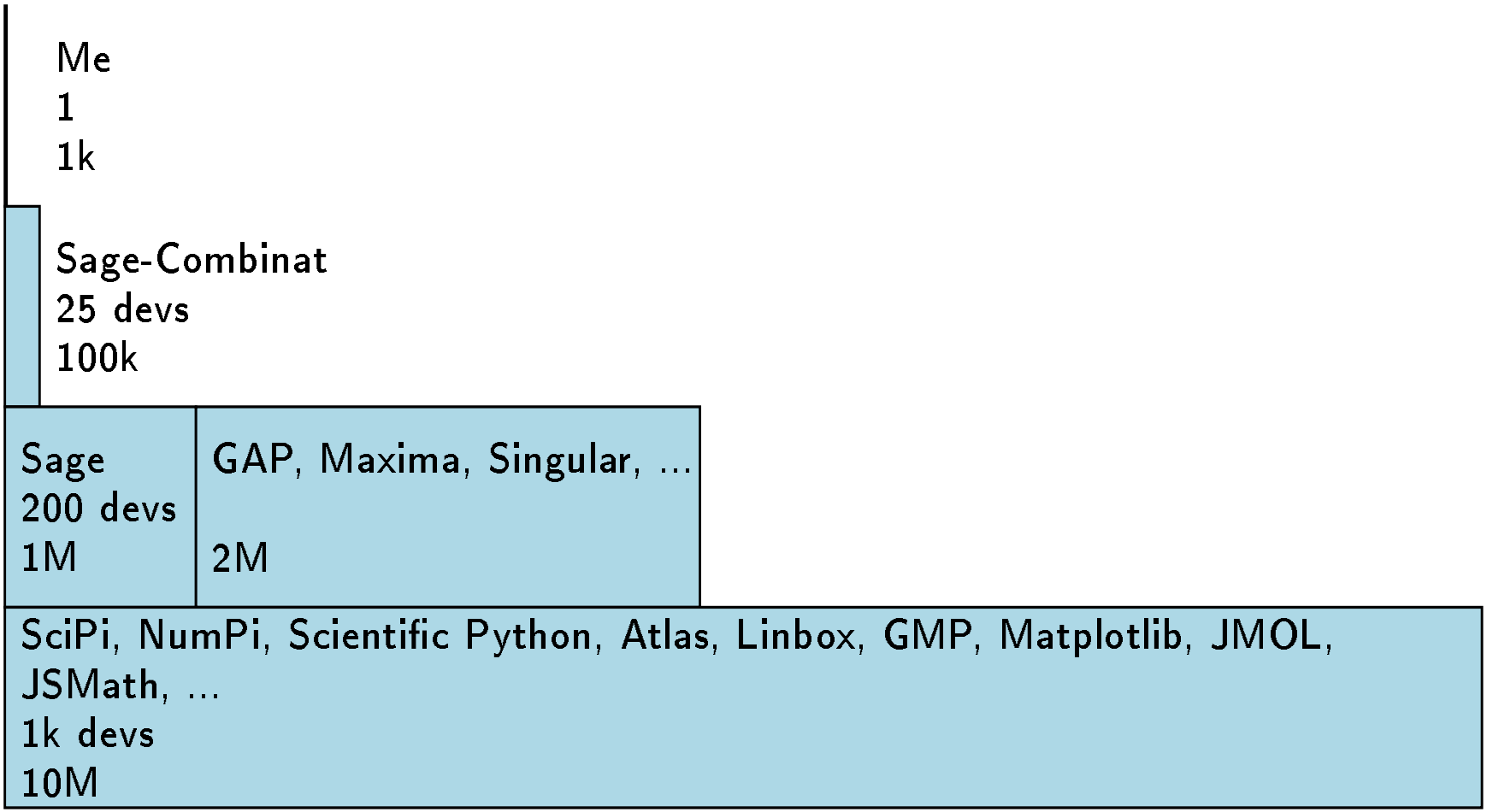
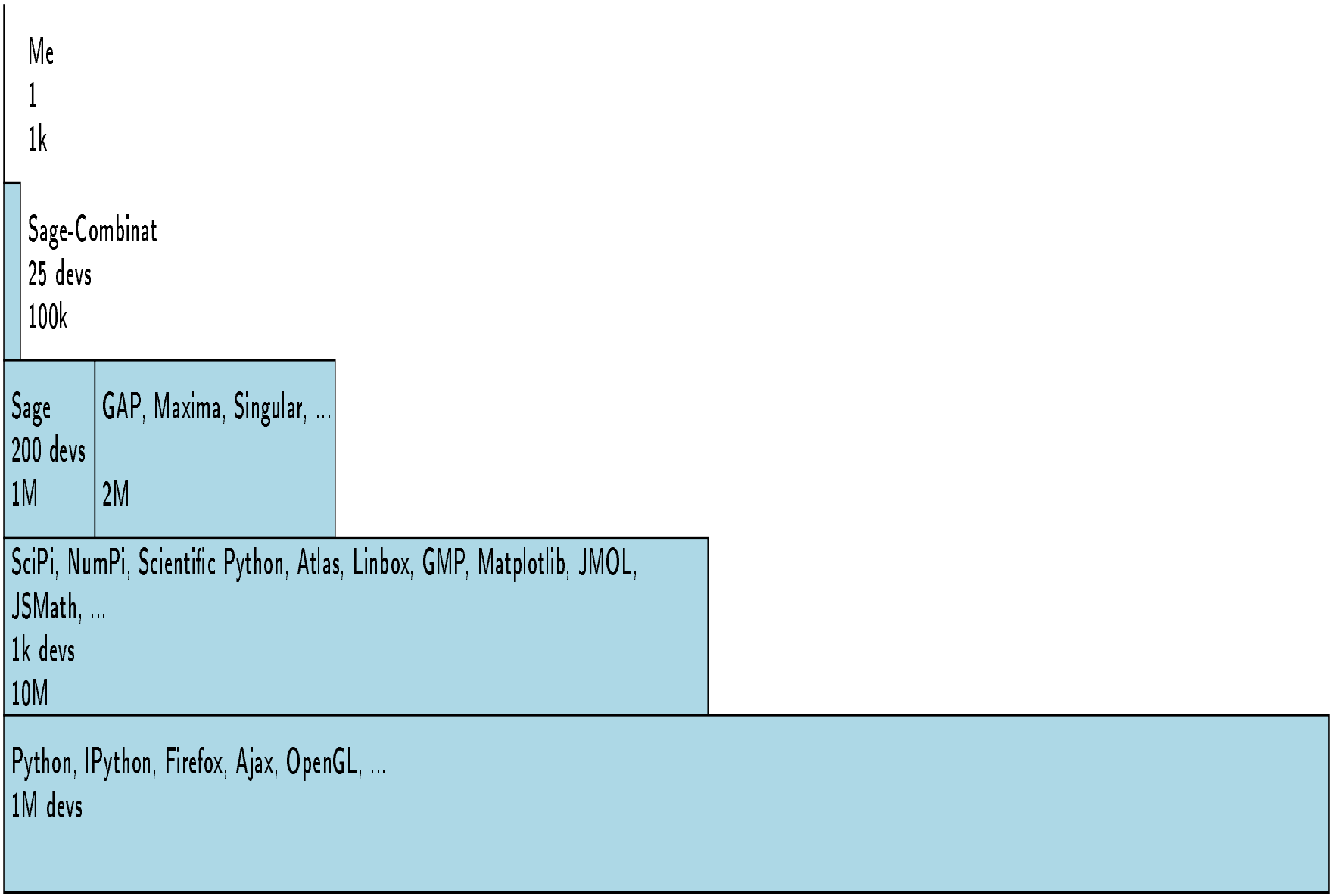
GAP, Maxima, Singular, ...
2M


Me
1
1k
Sage-Combinat
25 devs
100k
Sage
200 devs
1M
GAP, Maxima, Singular, ...
2M
SciPi, NumPi, Scientific Python, Atlas, Linbox, GMP, Matplotlib, JMOL, JSMath, ...
1k devs
10M


Me
1
1k
Sage-Combinat
25 devs
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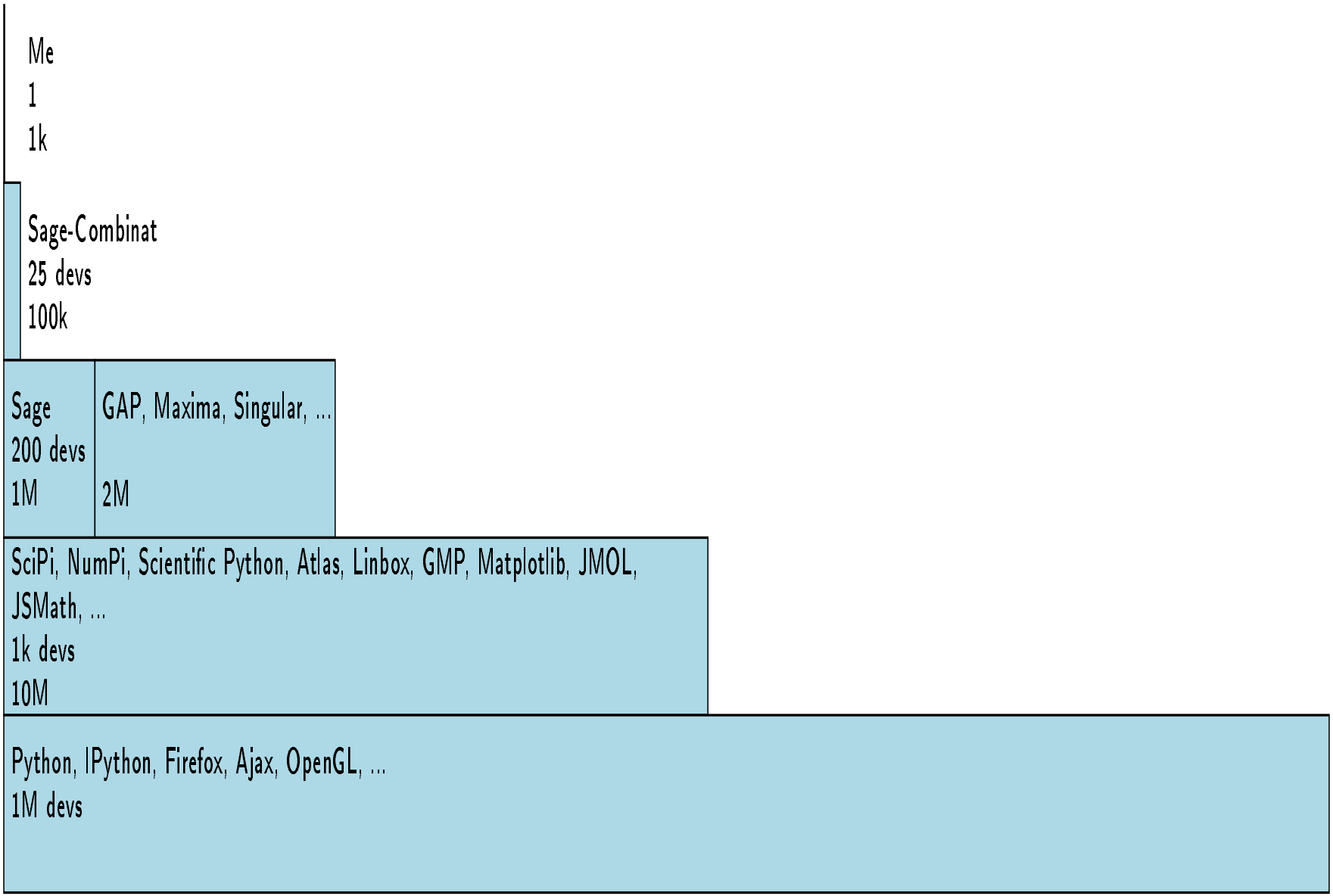
1M
GAP, Maxima, Singular, ...
2M
SciPi, NumPi, Scientific Python, Atlas, Linbox, GMP, Matplotlib, JMOL, JSMath, ...
1k devs
10M
Python, IPython, Firefox, Ajax, OpenGL, ...
1M devs

On the shoulders of a giant



Some useful features

LaTeX

In this LaTeX file, I typed:

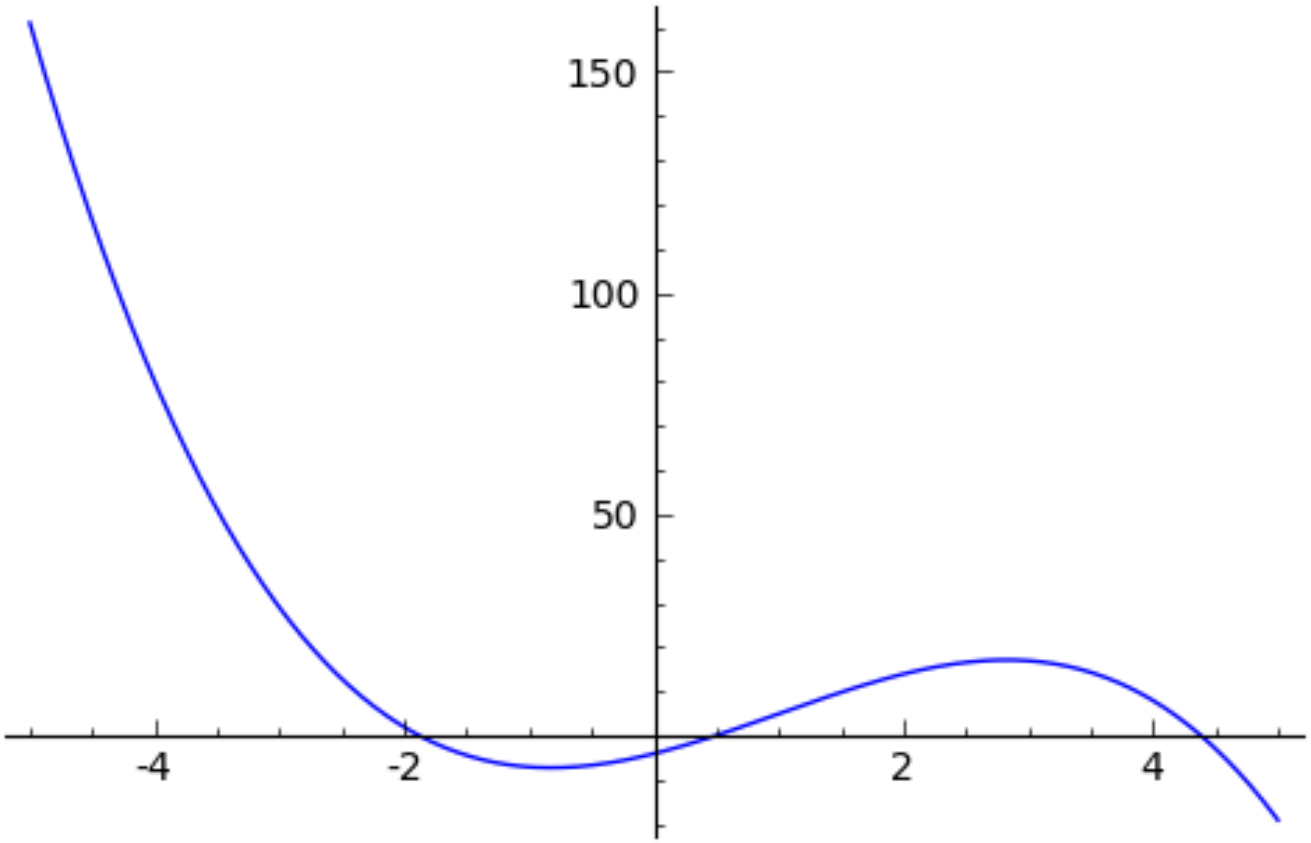
```
\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}
```

LaTeX

In this LaTeX file, I typed:

```
\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}
```

and it got replaced by:



LaTeX

In this LaTeX file:

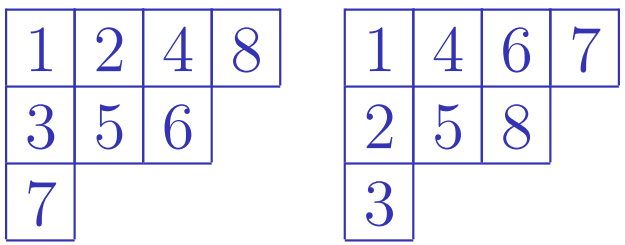
```
\begin{sagesilent}
  sigma = Permutation([7,3,1,5,2,6,8,4])
  P, Q = sigma.robinson_schensted()
\end{sagesilent}

Let $\sigma = \sage{sigma}$. The
Robinson-Schensted-Knuth algorithm produces
the tableaux:
    \[\sage{P} \quad \sage{Q}\]
```

LaTeX

It got replaced with:

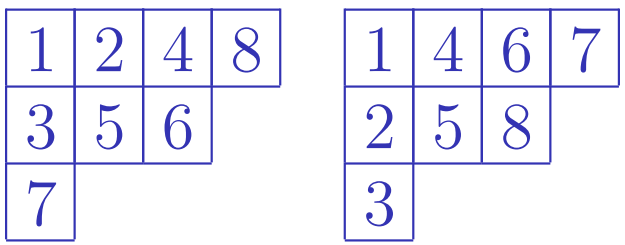
Let $\sigma =$ `[7, 3, 1, 5, 2, 6, 8, 4]`. The Robinson-Schensted-Knuth algorithm produces the tableaux:



LaTeX

It got replaced with:

Let $\sigma =$ `[7, 3, 1, 5, 2, 6, 8, 4]`. The Robinson-Schensted-Knuth algorithm produces the tableaux:



This is done with the *sagetex* package for LaTeX, written by Dan Drake. Of course, the package is included with Sage.