Quivers and Path Algebras

Sage Days 38: May 7–11, 2012

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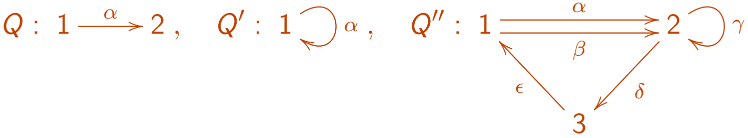
May 8th, 2012


Q : 1
α
2 ,
Q′ : 1
α ,
Q″ : 1
α
β
2
γ
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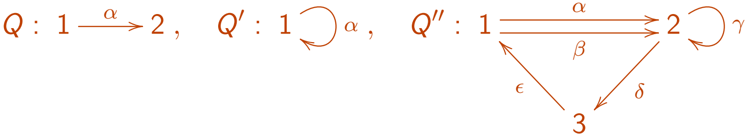
3

QPA: **Quivers**



$$Q = \begin{cases} Q_0, & \text{the set of vertices, usually } \{1, 2, \ldots, n\} \\ Q_1, & \text{the set of arrows} \\ \mathfrak{o}, \mathfrak{t} \colon Q_1 \to Q_0, & \text{origin/terminus vertex of an arrow} \end{cases}$$

QPA: **Representations (over $\mathbb{Q}$)**

$$Q:\ 1 \xrightarrow{\alpha} 2$$

$$M:\ \mathbb{Q}^2 \xrightarrow{\begin{bmatrix} 1 \\ -1 \end{bmatrix}} \mathbb{Q}\ ,\quad S_1:\ \mathbb{Q} \xrightarrow{[0]} 0\ ,\quad P_1:\ \mathbb{Q} \xrightarrow{[1]} \mathbb{Q}$$

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$$M = \begin{cases} M(i), & \text{finite dim'l vector space at vertex } i \in Q_0 \\ f_\alpha \colon M(i) \to M(j), & \text{linear map for each } \alpha \colon i \to j \in Q_1 \end{cases}$$
$$= (\{M(i)\}_{i \in Q_0}, \{f_\alpha\}_{\alpha \in Q_1})$$

Elements: $m = (m_1, m_2, \ldots, m_{|Q_0|}) \in M$ for $m_i \in M(i)$.

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Dimension vector:
$$\underline{\dim}(M) = (2,1)$$
$$\underline{\dim}(S_1) = (1,0)$$
$$\underline{\dim}(P_1) = (1,1)$$

QPA: **Direct sum**

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Direct sum:

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For $M = (\{M(i)\}_{i \in Q_0}, \{f_\alpha\}_{\alpha \in Q_1})$ and $N = (\{N(i)\}_{i \in Q_0}, \{g_\alpha\}_{\alpha \in Q_1})$

$$M \oplus N = \begin{cases} M(i) \oplus N(i), & i \in Q_0 \\ M(i) \oplus N(i) \xrightarrow{\left[\begin{smallmatrix} f_\alpha & 0 \\ 0 & g_\alpha \end{smallmatrix}\right]} M(j) \oplus N(j), & \alpha : i \to j \in Q_1 \end{cases}$$

QPA: **Isomorphisms**

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$$\begin{array}{lccc}
S_1 \oplus P_1 : & \mathbb{Q} \oplus \mathbb{Q} & \xrightarrow{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right]} & 0 \oplus \mathbb{Q} \\
\Big\downarrow \varphi & \Big\downarrow \left[\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right] & & \Big\downarrow \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] = [0\ 1]^{-1} \\
M : & \mathbb{Q}^2 & \xrightarrow{\left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right]} & \mathbb{Q}
\end{array}$$

$\left[\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right] [0\ 1]$, or equivalently $f_\alpha = \varphi(1) g_\alpha \varphi(2)^{-1}$.
Write: $M \simeq S_1 \oplus P_1$.

QPA: **Indecomposable representations**

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Krull-Remak-Schmidt-theorem:

(a) Any representation is isomorphic to a direct sum of indecomposable representations.

(b) Any decomposition into indecomposables is essentially unique.

QPA: **Homomorphisms**

$$M \simeq N \Leftrightarrow \exists\ \varphi(i) \colon M(i) \xrightarrow{\sim} N(i) \text{ such that } f_\alpha = \varphi(i) g_\alpha \varphi(j)^{-1}$$

$$\Leftrightarrow \quad \begin{array}{ccc} M(i) & \xrightarrow{\varphi(i)} & N(i) \\ {\scriptstyle f_\alpha}\downarrow & & \downarrow{\scriptstyle g_\alpha} \\ M(j) & \xrightarrow{\varphi(j)} & N(j) \end{array} \quad \text{commutes} \tag{1}$$

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$$S_1 \xrightarrow{\varphi} P_1 : \begin{array}{ccc} \mathbb{Q} & \xrightarrow{\varphi(1)} & \mathbb{Q} \\ \downarrow{\scriptstyle [0]} & & \downarrow{\scriptstyle [1]} \\ 0 & \xrightarrow{\varphi(2)} & \mathbb{Q} \end{array} \qquad P_1 \xrightarrow{\varphi} S_1 : \begin{array}{ccc} \mathbb{Q} & \xrightarrow{\varphi(1)} & \mathbb{Q} \\ \downarrow{\scriptstyle [1]} & & \downarrow{\scriptstyle [0]} \\ \mathbb{Q} & \xrightarrow{\varphi(2)} & 0 \end{array}$$

$$(\varphi(1), \varphi(2)) = (0, 0) \qquad \{(\varphi(1), \varphi(2))\} = \{(a, 0) \mid a \in \mathbb{Q}\}$$

$Q: 1 \circlearrowleft \alpha$, relation: α^3. In representations: $f_\alpha^3 = 0$.

QPA: **Quiver with relations**

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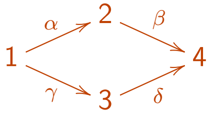
Q' : 

, relation: $3\alpha\beta - \gamma\delta$.

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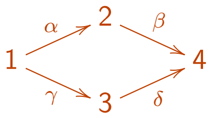

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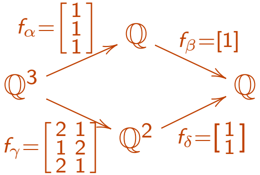
Representation:



with $3 f_\alpha f_\beta - f_\gamma f_\delta = 0$.

QPA: Path algebras (over $\mathbb{Q}$)

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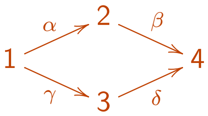

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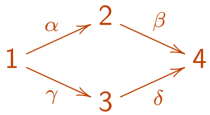

Basis of $\mathbb{Q}Q$: $\{e_1, e_2, e_3, e_4, \alpha, \beta, \gamma, \delta, \alpha\beta, \gamma\delta\}$ (e_i trivial paths)

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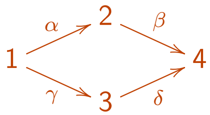

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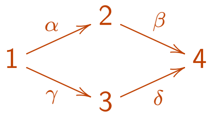

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Multiplicative structure: Induced by concatenation of paths in Q.

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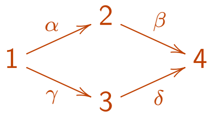

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Extend by distributivity: $(2e_1 + \alpha)(3\gamma + 4\beta) = 6\gamma + 4\alpha\beta$
Identity: $1_{\mathbb{Q}Q} = e_1 + e_2 + e_3 + e_4$

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$$\mathbb{Q}(\ 1 \circlearrowright \alpha\ ) \simeq \mathbb{Q}[x]$$

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$\rightsquigarrow$ a unique representation of elements in Λ can be computed algorithmically.

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Fact: Modules over Λ correspond to representations of Q satisfying the relations given by I.

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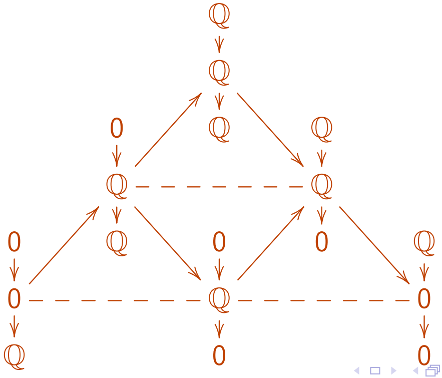
Almost split sequences: $\mathbb{Q}(\ 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3\ )$:



Some basic problems

Projective dimension: Q - quiver, ρ - admissible relations, M representation of Q satisfying ρ. Projective resolution of M:

$$\cdots \longrightarrow P(2) \xrightarrow{d_2} P(1) \xrightarrow{d_1} P(0) \xrightarrow{d_0} M \longrightarrow 0$$

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Finitistic dimension conjecture:

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Overall aim A software package and a common platform for development of software for finite dimensional quotients of path algebras. A package for GAP.

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Overall aim: A software package and a common platform for development of software for finite dimensional quotients of path algebras. A package for GAP.

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Current status: Quotients of path algebras, tensor products of algebras, representations (also projective/injective/simple), homomorphisms, Hom/End-spaces, radical/socle series, kernel/image/cokernel, pushout-pullback, projective covers, extensions of modules, almost split sequences, left/right approximations, (maximal) common summand, duality, transpose and more.

Design Inherited.

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Community http://sourceforge.net/projects/quiverspathalg/

QPA: Future projects

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- Representation of complexes of modules and basic operations on such
- Multiplicative structure of Ext-algebras of modules

QPA: Future projects

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QPA: Where to get it

Google: qpa quiver

http://sourceforge.net/projects/quiverspathalg/

http://www.math.ntnu.no/~oyvinso/QPA/