Geometric Engineering in Toric F-Theory

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Outline

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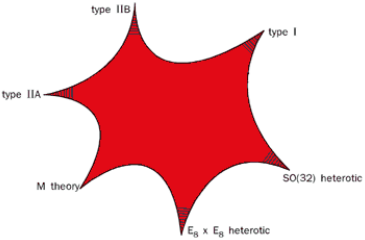
- Short introduction to F-Theory
- Geometric engineering in F-Theory

String dualities

Familiar picture of string dualities:

- Five 10d string theories with known microscopic descriptions
- 11d M-theory known only in its low-energy limit - 11d SUGRA
- String theories considered to be limits in certain corners of M-theory moduli space



Taken from Cern Courier.

$\Rightarrow$ Today we focus on M-theory and Type IIB and a way of thinking about them called F-Theory

What is F-Theory?

A convenient "definition" is by duality with M-theory:

- Compactify M-theory on $T^2 = S^1_a \times S^1_b$ with radii r_a, r_b
- In the limit $r_a \to 0$ one obtains Type IIA on S^1_b
- This setup is T-dual to Type IIB on $S^1_{b'}$ with radius $r_{b'} = \frac{l_s^2}{r_b}$
- Taking a second limit $r_b \to 0$ one ends up with decompactified Type IIB

What is F-Theory? (II)

One can generalize this procedure by considering manifolds with an elliptic fibration, i.e. one takes the compactification manifold Y_n to be

$$T^2 \to Y_n \to \mathcal{B}_{n-1} \tag{1}$$

for some $2n-2$-real-dimensional base manifold $\mathcal{B}$. Somewhat surprisingly, the duality carries over fiber-wise. In particular one often considers Calabi-Yau n-folds giving rise to minimal supersymmetry in $12-2n$ dimensions.

A geometrization of Type IIB

Okay - but what is the point of all of that?

$\Rightarrow$ F-theory geometrizes a certain Type IIB symmetry:

- When compactifying on Y_n, the moduli of Y_n become fields of the resulting effective theory
- Since $\mathrm{vol}(T^2) \to 0$, only the complex structure moduli of T^2 survive and become the axio-dilaton: $\tau = C_0 + \frac{i}{g_s}$
- $SL(2,\mathbb{Z})$ action on complex structure of T^2 is translated into $SL(2,\mathbb{Z})$ symmetry of Type IIB

In particular: Non-trivial fibrations give rise to varying axio-dilaton fields.

Gauge group from singularity resolution

Many more physical observables are realized as geometric quantities, since C_0 is sourced by $D7$-branes:

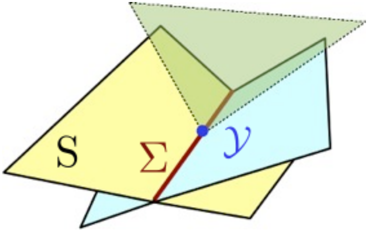
- $d \star dC_0 = \delta^{(2)}(z - z_0) \Rightarrow$ Codimension-1 singularities of T^2 associated with location of $D7$-branes.

There are subtleties in carrying out an M-theory reduction on a singular manifold. Singularities have to be resolved first and the intersection structure of the resolution divisors gives rise to the affine Dynkin diagram of the gauge algebra.

Dictionary between geometry and physics

This logic carries over to singularities in higher codimensions:

Codimension of Singularity	Physical Quantity
1	(Non-Abelian) gauge group
2	Matter curves
3	Matter couplings



Features of F-theory models

Phenomenological motivation:

- Can easily generate e.g. $SU(5)$ models with hypercharge flux breaking to Standard model gauge group
- Due to strong coupling effects, exceptional gauge symmetries can be realized $\Rightarrow$ allows certain phenomenologically desirable Yukawa couplings
- Existence of E_6 points leads to promising values of the CKM matrix

Mathematical motivation:

- Dualities between F-theory and heteroric string theory, mirror symmetry
- Study anomaly cancelation conditions $\Rightarrow$ geometric identities for Calabi-Yaus

Literature

For the original works on F-theory see for example papers by Beasley, Candelas, Donagi, Gukov, Heckman, Morrison, de la Ossa, Sen, Vafa, Witten, Wijnholt.

Over the past few years, there has been much interest in constructing both local and global F-theory compactifications with Abelian gauge factors. For some references, see for example papers by Braun, Cvetič, Dolan, Dudas, Grimm, Klevers, Marsano, Mayrhofer, Morrison, Palti, Park, Saulina, Schäfer-Nameki, Weigand.

In this talk, I wish to present the main results of the recent paper [arXiv:1306.0577] with Volker Braun and Thomas W. Grimm.

What did we study?

We looked at

- how to systematically construct n-folds with a given (toric) gauge symmetry
- which physical quantities are encoded by the Top (a certain building block of the reflexive polytope) alone
- under which conditions the fibration is flat

Roadmap

We wish to construct elliptically fibered Calabi-Yau n-folds Y_n with given

- Abelian gauge group
- Non-Abelian gauge group
- Base manifold

We assume that Y_n

- can be described as a complete intersection in a toric ambient space
- has an elliptic fiber described as a hypersurface in a two-dimensional ambient space

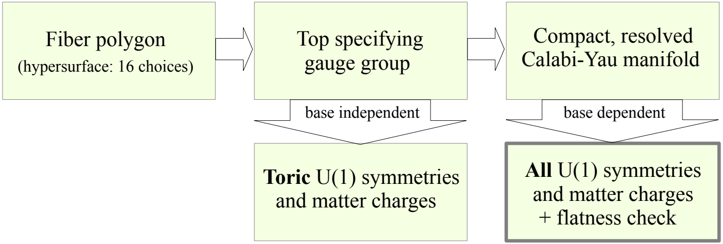
Additionally: How much information is already fixed before specifying the base manifold?

Scanning through reflexive polytopes

For small n, one might be able to scan through all models. Reflexive polytopes with reflexive sub-polygons give rise to elliptically fibered hypersurfaces. For $n = 3$ and $\mathcal{B} = \mathbb{P}^2$, see [Braun'11].

In particular, non-trivial gauge groups correspond to base loci over which the fiber becomes reducible $\Longleftrightarrow$ base rays with multiple pre-images under fan morphism corresponding to projection.

However, already for $n = 4$ this appears unfeasible. Instead proceed as follows:



Step I: Choosing a fiber

Choose one of 16 reflexive polygons F_i to embed the elliptic fiber in. This choice fixes the *minimum* number of $U(1)$s, i.e. it determines a subgroup

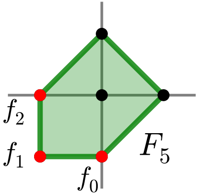
$$\mathrm{MW}_T \subseteq \mathrm{MW}\,. \tag{2}$$

Important: In general

$$\mathrm{rk}\,\mathrm{MW}_T \neq \text{number of toric sections of } F_i\,. \tag{3}$$

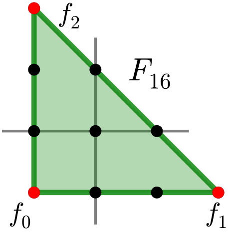
Fiber examples

We set $\sigma_i = f_i - f_0$, i.e. choose f_0 as zero section.



σ_1 and σ_2 independent

$\Rightarrow \ \mathrm{MW}_T = \mathbb{Z} \oplus \mathbb{Z}$



$3\sigma_1 = 0$, $2\sigma_1 = \sigma_2$

$\Rightarrow \ \mathrm{MW}_T = \mathbb{Z}_3$

What is a Top? (I)

We are interested in engineering specific non-Abelian gauge groups. The corresponding branes reside on divisors over which the fiber becomes reducible.

Let us therefore work *locally* and engineer the fiber over a certain base divisor. To do so we specify the preimage of the base ray under the fan morphism f giving rise to the projection $\pi : X \mapsto \mathcal{B}_{n-1}$ where X is the toric ambient space.

What is a Top? (II)

For a base ray $\mathbf{v}$ we call the lattice polytope

$$f^{-1}(\mathbf{v}) \cap P \tag{4}$$

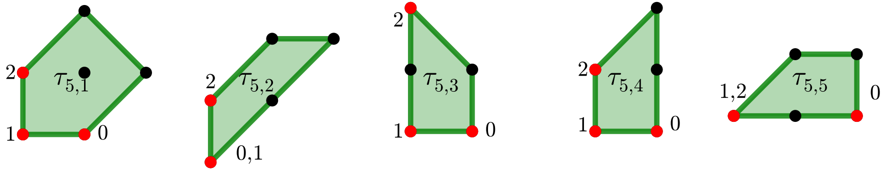
the Top over $\mathbf{v}$ where P is the reflexive polytope in which Y_n is embedded. Tops are three-dimensional. By definition, the fiber polygon is part of every Top. Tops are not reflexive; they serve as “building blocks” of our reflexive polytope.

Step II: Choosing a Top

The algorithm in [Bouchard,Skarke'03] allows to construct all possible Tops for a given non-Abelian gauge group.

Example

Modding out automorphisms, we find 5 different $SU(5)$ Tops for the fiber F_5 (dP_2):



$U(1)$ Charges

The choice of Top already determines the charge of the **10** and fixes the charges of the **5** representations modulo 5 for $SU(5)$.

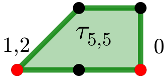
Example

Pick f_2 as zero section, f_0 and f_1 as generators for $U(1)_0$ and $U(1)_1$, respectively. Then

$$Q_{U(1)_0}(\mathbf{5}) \equiv 2 \mod 5 \qquad Q_{U(1)_1}(\mathbf{5}) \equiv 0 \mod 5 \tag{5}$$

$$Q_{U(1)_0}(\mathbf{10}) = -1 \qquad Q_{U(1)_1}(\mathbf{10}) = 0 \tag{6}$$

for the Top $\tau_{5,5}$:



Step III: Choosing a Base

Last of all, choose the base manifold with $\dim_{\mathbb{C}} \mathcal{B} = n - 1$.

Question: How can one classify and construct all possible reflexive lattice polytopes with given Top and base?

Answer: There exists a simple geometric algorithm.

The Algorithm

Let the fiber polygon have vertices $\mathbf{f}_1, \ldots, \mathbf{f}_r$, denote the base rays by $\mathbf{v}_1, \ldots, \mathbf{v}_s$ and place the non-Abelian singularity on $\mathbf{v}_1$. Take the Top vertices to be τ_j.
Embed into higher-dimensional polytope via

$$\mathbf{f}_i \mapsto (\mathbf{f}_i, \mathbf{0}), \quad \mathbf{v}_1 \mapsto (\tau_j, \mathbf{v}_1), \quad \mathbf{v}_i \mapsto (\mathbf{n}_i, \mathbf{v}_i) \ \text{ for } i \neq 1 \,. \tag{7}$$

The vectors $\mathbf{n}_i$ specify the embedding and $n-2$ of them can be set to zero to eliminate freedom in $GL(n-1, \mathbb{Z})$ transformations.
The convex hull of all points must not add additional points to the fiber polygon:
$\Rightarrow$ linear constraints for remaining $\mathbf{n}_i$

Polytope of Compactifications

The allowed values of $\mathbf{n}_i$ form the integral points of a lattice polytope.

Example

For $\tau_{5,5}$ with $\mathcal{B} = \mathbb{P}^3$, there are 30 inequivalent fourfolds

Flatness of the Fibration

If the fiber dimension varies, the fibration is called non-flat. Phenomenologically, one wants to avoid these cases, as they give rise to infinite towers of fields.

When does that happen? Consider the hypersurface equation restricted to the exceptional divisors

$$h|_{f_i=0} = \sum_{j}^{n_i} m_{j,fiber} p_{j,base} = 0 \tag{8}$$

where m is a fiber monomials and p is a base polynomial. Then one generically expects non-flat fibers in codimension $\min(\{n_i\})$ on the GUT divisor or $\min(\{n_i\}) + 1$ in the base manifold.

Way out: $\cap_j V(p_j) = \emptyset$ due to e.g. Stanley-Reisner ideal of $\mathcal{B}$.

Translating flatness conditions

Non-flat fibers have different origins depending on the codimension of the singular locus in the base.

- Codimension 2 (relevant for $n \geq 3$): Base *independent*, occur when Top has interior facet points
- Codimension > 2 (relevant for $n \geq 4$): Base *dependent*.

Requiring flatness for $n \geq 4$ imposes additional linear constraints on the $\mathbf{n}_i$ and is *non-generic* in this sense. In particular, certain combinations of Top and base are always non-flat.

Summary

- The choice of Top fixes the non-Abelian gauge group and the toric $U(1)$s, not, however, the non-toric $U(1)$s (see [Braun,Grimm,Keitel'13.02] for an example). A complete analysis of Abelian gauge groups is therefore *base dependent*.
- We have determined the *toric* Mordell-Weil group for all 16 reflexive fiber polygons and explained how to compute the *base-independent* **10** charge and **5** charge modulo 5.
- We gave an algorithm to construct all embeddings for a given Top and base and found geometric conditions for the flatness of the fibration.

Outlook

- In order to construct Y_n with multiple differently charged $\mathbf{10}$ curves, one needs to consider elliptic fibers in higher dimensional ambient spaces, e.g. $\mathbb{P}^3$. [Esole,Fullwood,Yau'11] [ongoing work]
- Many examples feature non-holomorphic zero sections inducing non-trivial KK-charge for the matter curves and leading to new CS-terms in M-theory compactification. [Grimm,Kapfer,Keitel '13.05]
- Improve current understanding of non-minimal singularities ⇔ non-flat fibers and their physics. [Lawrie,Schäfer-Nameki'12]
- Possibly tweak current algorithms and overcome last obstacles to set up a comprehensive scan of (a particular kind of) fourfolds?