

## Sage Quick Reference (Basic Math)

Peter Jipsen, version 1.1 (w/modification by nu)  
latest version at [wiki.sagemath.org/quickref](http://wiki.sagemath.org/quickref)  
GNU Free Document License, extend for your own use  
Aim: map standard math notation to Sage commands

## Notebook(とコマンドライン) Notebook (and commandline)

セルの評価: `(shift-enter)`

`com(tab)` *command* と補完しようとする。

`command?(tab)` ドキュメントを表示

`command??(tab)` ソースを表示

`a.(tab)` オブジェクト *a* のメソッドを表示 (more: `dir(a)`)

`search_doc('string or regexp')` ドキュメントへのリンク

`search_src('string or regexp')` ソースへのリンクを表示

`lprint()` LATEX 形式の出力に切替える

`version()` Sage のバージョンを表示

Insert cell: セルの間の青い線をクリック

Delete cell: 内容を消してから backspace

Evaluate cell: `(shift-enter)` ORGINAL TEXT

`com(tab)` tries to complete *command*

`command?(tab)` shows documentation

`command??(tab)` shows source

`a.(tab)` shows all methods for object *a* (more: `dir(a)`)

`search_doc('string or regexp')` shows links to docs

`search_src('string or regexp')` shows links to source

`lprint()` toggle LATEX output mode

`version()` print version of Sage

Insert cell: click on blue line between cells

Delete cell: delete content then backspace

## 数の型 Numerical types

整数:  $\mathbb{Z} = \text{ZZ}$  e.g. -2 -1 0 1 10^100

有理数:  $\mathbb{Q} = \text{QQ}$  e.g. 1/2 1/1000 314/100 -42

小数:  $\mathbb{R} \approx \text{RR}$  e.g. .5 0.001 3.14 -42

複素数:  $\mathbb{C} \approx \text{CC}$  e.g. 1+i 2.5-3\*i

ORGINAL TEXT

Integers:  $\mathbb{Z} = \text{ZZ}$  e.g. -2 -1 0 1 10^100

Rationals:  $\mathbb{Q} = \text{QQ}$  e.g. 1/2 1/1000 314/100 -42

Decimals:  $\mathbb{R} \approx \text{RR}$  e.g. .5 0.001 3.14 -42

Complex:  $\mathbb{C} \approx \text{CC}$  e.g. 1+i 2.5-3\*i

## 基本的な定数と函数 Basic constants and functions

定数:  $\pi = \text{pi}$   $e = \text{e}$   $i = \text{i}$   $\infty = \text{oo}$

近似値: `pi.n(digits=18)` = 3.14159265358979324

函数:  $\sin \cos \tan \sec \csc \cot \sinh \cosh \tanh \sech \csch$

$\coth \log \ln \exp$

$ab = \text{a}\ast\text{b}$   $\frac{a}{b} = \text{a}/\text{b}$   $a^b = \text{a}^\text{b}$   $\sqrt{x} = \text{sqrt}(\text{x})$

$\sqrt[n]{x} = \text{x}^(1/n)$   $|x| = \text{abs}(\text{x})$   $\log_b(x) = \text{log}(\text{x}, \text{b})$

不定元: e.g.  $t, u, v, y = \text{var('t u v y')}$

函数定義: e.g.  $f(x) = x^2$

(微分等ができるシンボリックな) 函数として: `f(x)=x^2`

Python 関数として定義する: `f=lambda x: x^2` または

`def f(x): return x^2`

..... ORGINAL TEXT

Constants:  $\pi = \text{pi}$   $e = \text{e}$   $i = \text{i}$   $\infty = \text{oo}$

Approximate: `pi.n(digits=18)` = 3.14159265358979324

Functions:  $\sin \cos \tan \sec \csc \cot \sinh \cosh \tanh \sech \csch$

$\coth \log \ln \exp$

$ab = \text{a}\ast\text{b}$   $\frac{a}{b} = \text{a}/\text{b}$   $a^b = \text{a}^\text{b}$   $\sqrt{x} = \text{sqrt}(\text{x})$

$\sqrt[n]{x} = \text{x}^(1/n)$   $|x| = \text{abs}(\text{x})$   $\log_b(x) = \text{log}(\text{x}, \text{b})$

Symbolic variables: e.g.  $t, u, v, y = \text{var('t u v y')}$

Define function: e.g.  $f(x) = x^2$

As symbolic function (can integrate, etc): `f(x)=x^2` or

As Python function: `f=lambda x: x^2` or

`def f(x): return x^2`

## 式に対する操作 Operations on expressions

`factor(...)` `expand(...)` `(...).simplify_...`

シンボリックな等式: `f(x)==g(x)`

`_` は直前の出力

`_+a` `_-a` `_*a` `_/_a` で等式を操作できる

$f(x) = g(x)$  を解く: `solve(f(x)==g(x), x)`

`solve([f(x,y)==0, g(x,y)==0], x, y)`

$x \in [a, b]$  s.t.  $f(x) \approx 0$  を探す: `find_root(f(x), a, b)`

$\sum_{i=k}^n f(i) = \text{sum}([\text{f}(i) \text{ for } i \text{ in } [\text{k..n}]])$

$\prod_{i=k}^n f(i) = \text{prod}([\text{f}(i) \text{ for } i \text{ in } [\text{k..n}]])$

..... ORGINAL TEXT

`factor(...)` `expand(...)` `(...).simplify_...`

Symbolic equations: `f(x)==g(x)`

`_` is previous output

`_+a` `_-a` `_*a` `_/_a` manipulates equation

Solve  $f(x) = g(x)$ : `solve(f(x)==g(x), x)`

`solve([f(x,y)==0, g(x,y)==0], x, y)`

`find_root(f(x), a, b)` find  $x \in [a, b]$  s.t.  $f(x) \approx 0$

$\sum_{i=k}^n f(i) = \text{sum}([\text{f}(i) \text{ for } i \text{ in } [\text{k..n}]])$

$\prod_{i=k}^n f(i) = \text{prod}([\text{f}(i) \text{ for } i \text{ in } [\text{k..n}]])$

## 微分積分 Calculus

$\lim_{x \rightarrow a} f(x) = \text{limit}(\text{f}(x), \text{x}=a)$

$\lim_{x \rightarrow a^-} f(x) = \text{limit}(\text{f}(x), \text{x}=a, \text{dir}='minus')$

$\lim_{x \rightarrow a^+} f(x) = \text{limit}(\text{f}(x), \text{x}=a, \text{dir}='plus')$

$\frac{d}{dx}(f(x)) = \text{diff}(\text{f}(x), \text{x})$

$\frac{\partial}{\partial x}(f(x, y)) = \text{diff}(\text{f}(x, y), \text{x})$

`diff` = `differentiate` = `derivative`

$\int f(x)dx = \text{integral}(\text{f}(x), \text{x})$

`integral` = `integrate`

$\int_a^b f(x)dx = \text{integral}(\text{f}(x), \text{x}, \text{a}, \text{b})$

次数  $n$  の  $a$  に関する Taylor 多項式: `taylor(f(x), x, a, n)` ORGINAL TEXT

$\lim_{x \rightarrow a} f(x) = \text{limit}(\text{f}(x), \text{x}=a)$

$\lim_{x \rightarrow a^-} f(x) = \text{limit}(\text{f}(x), \text{x}=a, \text{dir}='minus')$

$\lim_{x \rightarrow a^+} f(x) = \text{limit}(\text{f}(x), \text{x}=a, \text{dir}='plus')$

$\frac{d}{dx}(f(x)) = \text{diff}(\text{f}(x), \text{x})$

$\frac{\partial}{\partial x}(f(x, y)) = \text{diff}(\text{f}(x, y), \text{x})$

`diff` = `differentiate` = `derivative`

$\int f(x)dx = \text{integral}(\text{f}(x), \text{x})$

`integral` = `integrate`

$\int_a^b f(x)dx = \text{integral}(\text{f}(x), \text{x}, \text{a}, \text{b})$

Taylor polynomial, deg  $n$  about  $a$ : `taylor(f(x), x, a, n)`

## 二次元グラフィックス 2d graphics

`line([(x1, y1), ..., (xn, yn)], options)`

`polygon([(x1, y1), ..., (xn, yn)], options)`

`circle((x, y), r, options)`

`text("txt", (x, y), options)`

`options` は `plot.options` にあるものを使用,

例 `thickness=pixel, rgbcolor=(r, g, b), hue=h`  
(ただし  $0 \leq r, b, g, h \leq 1$ )

縦横比の調整には `figsize=[w, h]`

`plot(f(x), x_min, x_max, options)`

`parametric_plot((f(t), g(t)), t_min, t_max, options)`

`polar_plot(f(t), t_min, t_max, options)`

グラフの結合: `circle((1, 1), 1)+line([(0, 0), (2, 2)])`

`animate(list of graphics objects, options).show(delay=20)` ORGINAL TEXT

`line([(x1, y1), ..., (xn, yn)], options)`

`polygon([(x1, y1), ..., (xn, yn)], options)`

`circle((x, y), r, options)`

`text("txt", (x, y), options)`

`options` as in `plot.options`,

e.g. `thickness=pixel, rgbcolor=(r, g, b), hue=h`  
where  $0 \leq r, b, g, h \leq 1$

use option `figsize=[w, h]` to adjust aspect ratio

`plot(f(x), x_min, x_max, options)`

`parametric_plot((f(t), g(t)), t_min, t_max, options)`

`polar_plot(f(t), t_min, t_max, options)`

combine graphs: `circle((1, 1), 1)+line([(0, 0), (2, 2)])`

`animate(list of graphics objects, options).show(delay=20)`

## 三次元グラフィックス 3d graphics

**line3d**(( $x_1, y_1, z_1$ , ...,  $(x_n, y_n, z_n)$ ), options)  
**sphere**(( $x, y, z$ ), r, options)  
**tetrahedron**(( $x, y, z$ ), size, options)  
**cube**(( $x, y, z$ ), size, options)  
**octahedron**(( $x, y, z$ ), size, options)  
**dodecahedron**(( $x, y, z$ ), size, options)  
**icosahedron**(( $x, y, z$ ), size, options)  
**options の例** aspect\_ratio=[1,1,1] color='red' opacity  
**plot3d**(f( $x, y$ ), [x<sub>b</sub>, x<sub>e</sub>], [y<sub>b</sub>, y<sub>e</sub>], options)  
**オプションに** plot\_points=[m, n] or plot3d\_adaptive を使う  
**parametric\_plot3d**((f( $t$ ), g( $t$ ), h( $t$ )), [t<sub>b</sub>, t<sub>e</sub>], options)  
**parametric\_plot3d**((f( $u, v$ ), g( $u, v$ ), h( $u, v$ )), [u<sub>b</sub>, u<sub>e</sub>], [v<sub>b</sub>, v<sub>e</sub>], options)

graphics objects を結合するには + を使う

..... ORIGINAL TEXT

```

line3d(( $x_1, y_1, z_1$ , ...,  $(x_n, y_n, z_n)$ ), options)
sphere(( $x, y, z$ ), r, options)
tetrahedron(( $x, y, z$ ), size, options)
cube(( $x, y, z$ ), size, options)
octahedron(( $x, y, z$ ), size, options)
dodecahedron(( $x, y, z$ ), size, options)
icosahedron(( $x, y, z$ ), size, options)
options e.g. aspect_ratio=[1,1,1] color='red' opacity
plot3d(f( $x, y$ ), [xb, xe], [yb, ye], options)
add option plot_points=[m, n] or use plot3d_adaptive
parametric_plot3d((f( $t$ ), g( $t$ ), h( $t$ )), [tb, te], options)
parametric_plot3d((f( $u, v$ ), g( $u, v$ ), h( $u, v$ )), [ub, ue], [vb, ve], options)
use + to combine graphics objects
    
```

## 離散数学 Discrete math

$\lfloor x \rfloor = \text{floor}(x)$     $\lceil x \rceil = \text{ceil}(x)$   
 $n$  を  $k$  で割った余り = n%k    $k|n$  iff n%k==0  
 $n! = \text{factorial}(n)$     $\binom{x}{m} = \text{binomial}(x, m)$   
 $\phi = \text{golden_ratio}$     $\phi(n) = \text{euler_phi}(n)$   
**文字列:** 例 s = 'Hello' = "Hello" = "+"He"+'ll'o'  
 $s[0]='H'$     $s[-1]='o'$     $s[1:3]='el'$     $s[3:]='lo'$   
**リスト:** 例 [1, 'Hello', x] = []+[1, 'Hello']+[]  
**タプル:** 例 (1, 'Hello', x) (immutable)  
**集合:** 例 {1, 2, 1, a} = Set([1, 2, 1, 'a']) (= {1, 2, a})  
**集合の内包的記法** ≈ リストの内包表記, 例  
 $\{f(x) : x \in X, x > 0\} = \text{Set}([f(x) \text{ for } x \text{ in } X \text{ if } x > 0])$

..... ORIGINAL TEXT

```

 $\lfloor x \rfloor = \text{floor}(x)$     $\lceil x \rceil = \text{ceil}(x)$ 
Remainder of n divided by k = n%k    $k|n$  iff n%k==0
n! =  $\text{factorial}(n)$     $\binom{x}{m} = \text{binomial}(x, m)$ 
 $\phi = \text{golden_ratio}$     $\phi(n) = \text{euler_phi}(n)$ 
Strings: e.g. s = 'Hello' = "Hello" = "+"He"+'ll'o'
    
```

$s[0]='H'$     $s[-1]='o'$     $s[1:3]='el'$     $s[3:]='lo'$   
 Lists: e.g. [1, 'Hello', x] = []+[1, 'Hello']+[]  
 Tuples: e.g. (1, 'Hello', x) (immutable)  
 Sets: e.g. {1, 2, 1, a} = Set([1, 2, 1, 'a']) (= {1, 2, a})  
 List comprehension ≈ set builder notation, e.g.  
 $\{f(x) : x \in X, x > 0\} = \text{Set}([f(x) \text{ for } x \text{ in } X \text{ if } x > 0])$

## 線形代数 Linear algebra

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1, 2])$   
 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \text{matrix}([[1, 2], [3, 4]])$   
 $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \text{det}(\text{matrix}([[1, 2], [3, 4]]))$   
 $Av = A*v$     $A^{-1} = A^{-1}$     $A^t = A.\text{transpose}()$   
 methods: nrows() ncols() nullity() rank() trace()...

..... ORIGINAL TEXT

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1, 2])$   
 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \text{matrix}([[1, 2], [3, 4]])$   
 $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \text{det}(\text{matrix}([[1, 2], [3, 4]]))$   
 $Av = A*v$     $A^{-1} = A^{-1}$     $A^t = A.\text{transpose}()$   
 methods: nrows() ncols() nullity() rank() trace()...

..... ORIGINAL TEXT

## Sage のモジュールとパッケージ Sage modules and packages

**from module\_name import \*** (多くが既に読み込み済)  
**例** calculus coding combinat crypto functions games  
 geometry graphs groups logic matrix numerical plot  
 probability rings sets stats  
**sage.module\_name.all.(tab)** exportされたコマンドを表示  
 Std packages: Maxima GP/PARI GAP Singular R Shell...  
 Opt packages: Biopython Fricas(Axiom) Gnuplot Kash...  
**%package\_name** then use package command syntax  
**time command** timing informationを表示

..... ORIGINAL TEXT

**from module\_name import \*** (many preloaded)  
 e.g. calculus coding combinat crypto functions games geometry  
 graphs groups logic matrix numerical plot probability rings  
 sets stats  
**sage.module\_name.all.(tab)** shows exported commands  
 Std packages: Maxima GP/PARI GAP Singular R Shell...  
 Opt packages: Biopython Fricas(Axiom) Gnuplot Kash...  
**%package\_name** then use package command syntax  
**time command** to show timing information