Algorithms in Number Theory Project

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Let K be a number field, $\mathfrak{o}$ its ring of integers, $\Omega(K)$ the set of prime spots in K, and for $a, b \in \mathfrak{o}$ and $\nu \in \Omega(K)$, let $(a,b)_\nu$ denote the Hilbert symbol. In his PhD Thesis, Markus Kirschmer gives several algorithms for computing global objects from a set of local invariants. The first, Algorithm 3.4.1, returns $a \in \mathfrak{o}$ such that for a given subset $S \subseteq \Omega(K)$ of even cardinality and $b \in K^\times$, we have

$$\{\nu \in \Omega(K) : (a,b)_\nu = -1\} = S.$$

The steps for this algorithm are given in the paper and they involve group and module theory. It would be helpful for what follows to implement this in SAGE. The second, Algorithm 3.4.3, uses the 3.4.1 to compute a global quadratic form with a prescribed set of local conditions. This algorithm might require some more explicit understanding of quadratic forms. This algorithm has garnered a high degree of interest from people studying quadratic forms. Both of these projects are well suited for an experienced coder who is interested in learning the ins-and-outs of the process of publishing work to SAGE.